TS Inter Maths 1A - 4 Marks Important Questions

Matrices

1. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then show that $(aI + bE)^3 = a^3I + 3a^2bE$
2. If $\theta - \emptyset = \frac{\pi}{2}$, then show that $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\emptyset & \cos\emptyset\sin\emptyset \\ \cos\emptyset\sin\emptyset & \sin^2\emptyset \end{bmatrix} = 0$
3. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then for any integer $n \geq 1$show that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$
4. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then show that ∀ positive integers $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$
5. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then show that $A^2 - 4A - 5I = 0$
6. If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$ then find $A + A^T$, $A.A^T$
7. If $A = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -3 & 0 \\ 5 & 4 \end{bmatrix}$then verify $(AB)^| = B^|A^|$
8. If $A = \begin{bmatrix} 7 & -2 \\ -1 & 2 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 4 & 2 \\ -1 & 0 \end{bmatrix}$then find $AB^|$ and $BA^|$
9. If $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 4 & 0 \\ 4 & -2 & -1 \end{bmatrix}$ then prove that $(A + B)^T = A^T + B^T$
10. Find the value of x if $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$
11. Show that $\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix} = (a-b)(b-c)(c-a)$
12. Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$
13. Show that $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = 4xyz$
14. If $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$then find the adjoint and inverse of A
15. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$then find A^{-1}

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16. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then show that $A^{-1} = A^3$

17. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ then show that adjoint A =3 $A^{|}$. Find A^{-1}

18. If A and B are invertible then show that AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$

19. Show that the determinant of skew symmetric matrix of order 3 is alway2s zero

Addition of Vectors

1. Let ABCDEF be a regular hexagon with center O. Show that
$\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\,\overline{AD} = 6\,\overline{AO}$

2. In ΔABC if O is the circum Centre and H is the orthocenter, then shoe that
(i) $\overline{AO} + \overline{OB} + \overline{OC} = \overline{OH}$ (ii) $\overline{HA} + \overline{HB} + \overline{HC} = 2\overline{HO}$

3. In the two-dimensional plane, prove by using vector method the equation of the line whose intercepts are a and b is $\frac{x}{a} + \frac{y}{b} = 1$

4. $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar vectors. Prove that the following four points are coplanar
$-\bar{a} + 4\bar{b} - 3\bar{c}, 3\bar{a} + 2\bar{b} - 5\bar{c}, -3\bar{a} + 8\,\bar{b} - 5\bar{c}, -3\bar{a} + 2\bar{b} + \bar{c}$

5. $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar vectors. Prove that the following four points are coplanar
$6\bar{a} + 2\bar{b} - \bar{c}, 2\bar{a} - \bar{b} + 3\bar{c}, -\bar{a} + 2\,\bar{b} - 4\bar{c}, -12\bar{a} - \bar{b} - 3\bar{c}$

6. If the points whose position vectors are $3\bar{i} - 2\,\bar{j} - \bar{k}, 2\bar{i} + 3\,\bar{j} - 4\bar{k}, -\bar{i} + \bar{j} + 2\,\bar{k}$ and
$4\bar{i} + 5\,\bar{j} + \lambda\,\bar{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$

7. If $\bar{i}, \bar{j}, \bar{k}$ are unit vectors along the positive direction of the coordinate axes, then show that the four points are $3\bar{i} - 2\,\bar{j} - \bar{k}, 2\bar{i} + 3\,\bar{j} - 4\bar{k}, -\bar{i} + \bar{j} + 2\,\bar{k}$ and $4\bar{i} + 5\,\bar{j} + \lambda\,\bar{k}$ are coplanar

8. Find the vector equation of the plane passing through the points $4\bar{i} - 3\,\bar{j} - \bar{k}, 3\bar{i} + 7\,\bar{j} - 10\bar{k}$, and
$2\bar{i} + 5\,\bar{j} - 7\bar{k}$ and show that the point $\bar{i} + 2\,\bar{j} - 3\bar{k}$ lies in the plane.

9. Show that the line joining the pair of points $6\bar{a} - 4\bar{b} + 4\bar{c}, -4\bar{c}$ and the line joining the points
$-\bar{a} - 2\bar{b} - 3\bar{c}, \bar{a} + 2\bar{b} - 5\bar{c}$ intersects at the point $-4\bar{c}$ when $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar vectors.

10. If $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar find the point of intersection of the line passing through the points $2\bar{a} + 3\bar{b} - \bar{c}, 3\bar{a} + 4\bar{b} - 2\bar{c}$ with the line joining the points $\bar{a} - 2\,\bar{b} + 3\bar{c}, \bar{a} - 6\bar{b} + 6\bar{c}$

11. If $\bar{a} + \bar{b} + \bar{c} = \alpha\bar{d}$, $\bar{b} + \bar{c} + \bar{d} = \beta\bar{a}$ and $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar vectors, then show that,
$\bar{a} + \bar{b} + \bar{c} + \bar{d} = 0$.

12. Let $\bar{a}, \bar{b}$ are non – collinear vectors. If $\bar{\alpha} = (x + 4y)\,\bar{a} + (2x + y + 1)\bar{b}$ and
$\bar{\beta} = (y - 2x + 2)\,\bar{a} + (2x - 3y - 1)\bar{b}$ are such that $3\bar{\alpha} = 2\,\bar{\beta}$ then find x and y.

13. If $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar vectors, then test for the collinearity of the points $3\bar{a} - 4\bar{b} + 3\bar{c}$,
$-4\bar{a} + 5\bar{b} - 6\bar{c}, 4\bar{a} - 7\,\bar{b} + 6\bar{c}$.

Product of Vectors

1. Prove that the smaller angle θ between any two diagonals of a cube is given by $\cos\theta = 1/3$. Find unit vector perpendicular to the plane passing through the points A (1, 2, 3), B (2, 3, 1) and C (3, 1, 2).
2. Find the unit vector perpendicular to the plane passing through the points (1, 2, 3), (2, – 1, 1) and (1, 2, – 4).
3. Find the unit vector perpendicular to the plane determined by the points P (1, – 1, 2), Q (2, 0, – 1) and R = (0, 2, 1).
4. Find the volume of tetrahedron whose vertices are (1, 2, 1), (3, 2, 5), (2, – 1, 0) and (– 1, 0, 1).
5. Find the volume of tetrahedron having edges $\bar{i} + \bar{j} + \bar{k}$, $\bar{i} - \bar{j}$ and $\bar{i} + 2\bar{j} + \bar{k}$.
6. Find the volume of parallelepiped whose contaminate edges are represented by the vectors $2\bar{i} - 3\bar{j} + \bar{k}$, $\bar{i} - \bar{j} + 2\bar{k}$ and $2\bar{i} + \bar{j} - \bar{k}$.
7. Determine λ, for which the volume of parallelepiped having contaminate edges $\bar{i} + \bar{j}$, $3\bar{i} - \bar{j}$ and $3\bar{j} + \lambda\bar{k}$.
8. If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}$, $\bar{b} = \bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} - \bar{j} + \bar{k}$, then compute $\bar{a} \times (\bar{b} \times \bar{c})$ and verify that it is perpendicular to $\bar{a}$.
9. If $\bar{a} = \bar{i} - 2\bar{j} - 3\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} + 3\bar{j} - 2\bar{k}$, then verify that $\bar{a} \times (\bar{b} \times \bar{c}) \neq (\bar{a} \times \bar{b}) \times \bar{c}$
10. If $\bar{a} + \bar{b} + \bar{c} = 0$, $|\bar{a}| = 3$, $|\bar{b}| = 5$ and $|\bar{c}| = 7$ then find the angle between $\bar{a}$ and $\bar{b}$
11. Let $\bar{a} = 4\bar{i} + 5\bar{j} - \bar{k}$, $\bar{b} = \bar{i} - 4\bar{j} + 5\bar{k}$ and $\bar{c} = 3\bar{i} + \bar{j} - \bar{k}$. Find the vector which is perpendicular to both $\bar{a}$ and $\bar{b}$ whose magnitude is twenty one times the magnitude $\bar{c}$
12. Find λ, in order that the four points A (3, 2, 1), B (4, λ, 5), C (4, 2, – 2) and D (6, 5, – 1) be coplanar.
13. If $\bar{a} = 2\bar{i} + \bar{j} - \bar{k}$, $\bar{b} = -\bar{i} + 2\bar{j} - 4\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$, then find $(\bar{a} \times \bar{b}) \cdot (\bar{b} \times \bar{c})$.
14. If $\bar{a} = 3\bar{i} - \bar{j} + 2\bar{k}$, $\bar{b} = -\bar{i} + 3\bar{j} + 2\bar{k}$, $\bar{c} = 4\bar{i} + 5\bar{j} - 2\bar{k}$ and $\bar{d} = \bar{i} + 3\bar{j} + 5\bar{k}$, then find the following: (i) $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})$ (ii) $(\bar{a} \times \bar{b}) \cdot \bar{c} - (\bar{a} \times \bar{d}) \cdot \bar{b}$
15. Show that the angle in a semicircle is right angle.

Trigonometric Ratios Up To Transformations

1. Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$
2. If A is not multiple of $\frac{\pi}{2}$, then prove that (i) tan A + cot A = 2 cosec 2A
   (ii) cot A – tan A = 2cot 2A
3. Let ABC be a triangle such that cot A + cot B + cot C = $\sqrt{3}$, then prove that ΔABC is an equilateral triangle.
4. Prove that $\tan 70^0 - \tan 20^0 = 2\tan 50^0$.
5. Prove that $\sin\frac{\pi}{5}\ \sin\frac{2\pi}{5}\ \sin\frac{3\pi}{5}\ \sin\frac{4\pi}{5} = \frac{5}{16}$.
6. If $A + B = 45^0$, then prove that (i) (1 + tan A) (1 + tan B) = 2
   (ii) (cot A – 1) (cot B – 1) = 2
7. If $A + B = \frac{3\pi}{4}$, then prove that (1 – tan A) (1+ tan B) = 2
8. Prove that $\left(1 + \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)\left(1 + \cos\frac{7\pi}{10}\right)\left(1 + \cos\frac{9\pi}{10}\right) = \frac{1}{16}$

9. Prove that $\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}\cdot\cos\frac{8\pi}{7} = \frac{1}{8}$.

10. Prove that $\cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{3\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{5\pi}{11} = \frac{1}{32}$.

11. If $0 < A < B < \frac{\pi}{4}$ $\sin(A+B) = \frac{24}{25}$, $\cos(A-B) = \frac{4}{5}$ then find the value of tan 2A.

12. Prove that $\sqrt{3}\,\mathrm{cosec}\,20^0 - \sec 20^0 = 4$

13. Prove that $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$

14. Prove that $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = \frac{3}{2}$

15. If 3A is not an odd multiple of $\frac{\pi}{2}$, prove that $\tan A \tan(60-A)\tan(60+A) = \tan 3A$ and hence find the value of $\tan 6^0 \tan 42^0 \tan 66^0 \tan 78^0$.

16. If A is not an integral multiple of π prove that $\cos A\ \cos 2A\ \cos 4A\ \cos 8A = \frac{\sin 16A}{16\sin A}$ and hence deduce that cos

17. $\sin A \sin(60-A)\sin(60+A) = \frac{1}{4}\sin 3A$ and hence deduce that $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$

18. For A ∈ R, prove that $\cos A \cos(60-A)\cos(60+A) = \frac{1}{4}\cos 3A$ and hence deduce that $\cos\frac{\pi}{9}\ \cos\frac{2\pi}{9}\cos\frac{3\pi}{9}\cos\frac{4\pi}{9} = \frac{1}{16}$.

19. If none of the denominators is $\frac{2\pi}{15}\ \cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15} = \frac{1}{16}$.

20. For A ∈ R, prove that zero, then prove that

$$\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n + \left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n = \begin{cases} 2\cot^n\left(\frac{A-B}{2}\right) \text{ if n is even} \\ 0 \text{ if n is odd}\end{cases}$$

21. If $\sec(\theta+\alpha) + \sec(\theta-\alpha) = 2\sec\theta$ and $\cos\alpha \neq 1$, then show that $\cos\theta = \pm\sqrt{2}\cos\frac{\alpha}{2}$

Trigonometric Equations

1. Solve the equation $\sqrt{3}\sin\theta - \cos\theta$
2. Find the value of x in $(-\pi, \pi)$ satisfying the equation $8^{1+\cos x+\cos^2 x+\cdots} = 4^3$
3. If θ_1, θ_2 are solution of the equation $a\cos 2\theta + b\sin\theta + = c$, $\tan\theta_1 \neq \tan\theta_2$ and $a + c \neq 0$ then find the values of
   (i) $\tan\theta_1 + \tan\theta_2$ (ii) $\tan\theta_1\cdot\tan\theta_2$ (ii) $\tan(\theta_1+\theta_2)$
4. Solve and write the general solution of the equation $2\cos^2\theta - \sqrt{3}\sin\theta + 1 = 0$
5. Solve and write the general solution of the equation $\sqrt{2}(\sin x + \cos x) = \sqrt{3}$
6. Solve and write the general solution of the equation $\tan\theta + \cot\theta + 1 = 5\sec\theta$
7. Solve $1 + \sin^2\theta = 3\sin\theta\cos\theta$
8. Solve $\sin x + \sqrt{3}\ \cos x = \sqrt{2}$
9. Solve the equation $\cot^2 x - (\sqrt{3}+1)\cot x + \sqrt{3} = 0$ $(0 < x < \frac{\pi}{2})$
10. Solve $\sin 2x - \cos 2x = \sin x - \cos x$
11. Solve $4\sin x\ \sin 2x\ \sin 4x = \sin 3x$
12. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then prove that $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$

13. Given p ≠ ± q, show that the solution of cos p θ + cos q θ from two series each of which in A.P. Also find the common difference of A.P.
14. Given tan p θ + cot q θ and p ≠ ± q, show that the solutions are in A.P. with common difference $\frac{\pi}{a+b}$
15. Solve and write the general solution of the equation $4\cos^2\theta + \sqrt{3} = 2(\sqrt{3}+1)\cos\theta$
16. Solve $\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$ $(0 \le x \le 2\pi)$
17. Solve $\tan\theta + \sec\theta = \sqrt{3}$ $(0 \le \theta \le 2\pi)$
18. Solve $\sin 3\alpha = 4\sin\alpha \sin(x+\alpha)\sin(x-\alpha)$ where $\alpha \neq n\pi, n \in Z$

Inverse Trigonometric Functions

1. Prove that $\mathrm{Tan}^{-1}\frac{1}{2} + \mathrm{Tan}^{-1}\frac{1}{5} + \mathrm{Tan}^{-1}\frac{1}{8} = \frac{\pi}{4}$
2. Find the value of $\tan\left[\mathrm{Cos}^{-1}\frac{4}{5} + \mathrm{Tan}^{-1}\frac{2}{3}\right]$
3. Solve the equation $3\,\mathrm{Sin}^{-1}\frac{2x}{1+x^2} - 4\,\mathrm{Cos}^{-1}\frac{1-x^2}{1+x^2} + 2\,\mathrm{Tan}^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$
4. If $\mathrm{Sin}^{-1}x + \mathrm{Sin}^{-1}y + \mathrm{Sin}^{-1}z = \pi$, then prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$
5. Prove that $\mathrm{Sin}^{-1}\frac{4}{5} + \mathrm{Sin}^{-1}\frac{7}{25} = \mathrm{Sin}^{-1}\frac{117}{125}$
6. Prove that $\mathrm{Sin}^{-1}\frac{3}{5} + \mathrm{Sin}^{-1}\frac{8}{17} = \mathrm{Cos}^{-1}\frac{36}{85}$
7. Prove that $\mathrm{Sin}^{-1}\frac{4}{5} + \mathrm{Tan}^{-1}\frac{1}{3} = \frac{\pi}{2}$
8. Prove that $\mathrm{Sin}^{-1}\frac{4}{5} + \mathrm{Sin}^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$
9. Show that $\cot\left(\mathrm{Sin}^{-1}\sqrt{\frac{13}{17}}\right) = \sin\left(\mathrm{Tan}^{-1}\frac{2}{3}\right)$
10. Prove that $\mathrm{Cos}^{-1}\frac{4}{5} + \mathrm{Sin}^{-1}\frac{3}{\sqrt{34}} = \mathrm{Tan}^{-1}\frac{27}{11}$
11. Prove that $\sin\left[\mathrm{Cot}^{-1}\frac{2x}{1-x^2} + \mathrm{Cos}^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 1$
12. Show that $\sec^2(\mathrm{Tan}^{-1}2) + \mathrm{cosec}^2(\mathrm{Cot}^{-1}2) = 10$
13. If $\mathrm{Cos}^{-1}p + \mathrm{Cos}^{-1}q + \mathrm{Cos}^{-1}r = \pi$, then prove that $p^2 + q^2 + r^2 + 2pqr = 1$
14. If $\mathrm{Tan}^{-1}x + \mathrm{Tan}^{-1}y + \mathrm{Tan}^{-1}z = \pi$, then prove that $x + y + z = xyz$
15. If $\mathrm{Tan}^{-1}x + \mathrm{Tan}^{-1}y + \mathrm{Tan}^{-1}z = \frac{\pi}{2}$, then prove that $xy + yz + zx = 1$
16. If $\mathrm{Cos}^{-1}\frac{p}{a} + \mathrm{Cos}^{-1}\frac{q}{b} = \alpha$, then prove that $\frac{p^2}{q^2} - \frac{2pq}{ab} + \frac{q^2}{b^2} = \sin^2\alpha$
17. Solve the equation $\mathrm{Tan}^{-1}\frac{x-1}{x-2} + \mathrm{Tan}^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$
18. Prove that $\cos[\mathrm{Tan}^{-1}\{\sin(\mathrm{Cot}^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$
19. Prove that $\mathrm{Tan}^{-1}\frac{3}{4} + \mathrm{Tan}^{-1}\frac{3}{5} - \mathrm{Tan}^{-1}\frac{8}{19} = \frac{\pi}{4}$
20. Solve $\mathrm{Sin}^{-1}x + \mathrm{Sin}^{-1}2x = \frac{\pi}{3}$

Properties of Triangles

1. Show that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$

2. Show that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \frac{s^2}{\Delta}$

3. If $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = 3 : 5 : 7$, then show that $a : b : c = 6 : 5 : 4$

4. Show that in Δ ABC, $a = b\cos C + c\cos B$

5. Show that in Δ ABC, $\tan\left(\frac{B - C}{2}\right) = \frac{b - c}{b + c}\cot\frac{A}{2}$

6. If $C = 60^0$, then show that (i) $\frac{a}{b + c} + \frac{a}{b + c} = 1$ (ii) $\frac{b}{c^2 - a^2} + \frac{a}{c^2 - b^2} = 0$

7. In Δ ABC, if $\frac{1}{a + c} + \frac{1}{b + c} = \frac{3}{a + b + c}$, then show that $C = 60^0$

8. If $a = (b - c)\sec\theta$, then prove that $\tan\theta = \frac{2\sqrt{bc}}{b - c}\sin\frac{A}{2}$

9. If $a = (b + c)\cos\theta$, then prove that $\tan\theta = \frac{2\sqrt{bc}}{b + c}\cos\frac{A}{2}$

10. If $\sin\theta = \frac{a}{b + c}$, then prove that $\cos\theta = \frac{2\sqrt{bc}}{b + c}\cos\frac{A}{2}$

11. In Δ ABC, show that $\tan\frac{b^2 - c^2}{a^2} = \frac{\sin(B - C)}{\sin(B + C)}$

12. Show that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$

13. Show that $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

14. Show that $\frac{1}{r^2} + \frac{1}{{r_1}^2} + \frac{1}{{r_2}^2} + \frac{1}{{r_3}^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$

15. Show that $(b - c)^2 \cos^2\frac{A}{2} + (b + c)^2 \sin^2\frac{A}{2} = a^2$

16. Show that $a^2\cot A + b^2\cot B + c^2\cot C = \frac{abc}{R}$

17. If $a : b : c = 7 : 8 : 9$, then find $\cos A : \text{Cos } B : \text{Cos } C$

18. If $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in AP, then prove that a, b, c are in AP

19. Show that $b^2\sin 2C + c^2\sin 2B = 2bc\sin A$

20. If $(r_2 - r_1)(r_3 - r_1) = 2r_1 r_3$, then show that $A = 90^0$