

TS Inter Maths 1B - 4 Marks Important Questions

Locus

1. Find the equation of locus of P, if the ratio of the distance from P to (5, - 4) and (7, 6) is 2 : 3.
2. A (1, 2), B (2, - 3) and (- 2, 3) are three points. A point P moves such that $PA^2 + PB^2 = 2 PC^2$
3. Find the equation of locus of point P such that $PA^2 + PB^2 = 2c^2$ where $A = (a, 0)$, $B = (- a, 0)$ and $0 < |a| < |c|$
4. A (5, 3) and B (3, - 2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq. units.
5. A (2, 3) and B (- 3, 4) are two given points. Find the equation of locus of P, so that the area of triangle PAB is 8.5 sq. units.
6. Find the equation of locus of a point, the difference of whose distances from (- 5, 0) and (5, 0) is 8.
7. Find the equation of locus of a point, the sum of whose distances from (0, 2) and (0, -2) is 6 units.
8. Find the equation of locus of P, if $A = (2, 3)$, $B = (2, - 3)$ and $PA + PB = 8$.
9. Find the equation of locus of P, if $A = (4, 0)$, $B = (- 4, 0)$ and $|PA - PB| = 4$.
10. Find the equation of locus of P, if the line segment joining (2, 3) and (- 1, 5) subtends a right angle at P.
11. The ends of the hypotenuse of a right angled triangle are (0, 6) and (6, 0). Find the equation of locus its third vertex.
12. Find the locus of the third vertex of a right angled triangle, the ends of whose hypotenuse are (4, 0) and (0, 4).
13. Find the equation of locus of a point P such that the distance of P from the origin is twice the distance of P from A (1, 2).
14. Find the equation of locus of a point P, if the distance of P from A (3, 0) is twice the distance of P from (- 3, 0).

Transformation of Axes

1. When the origin shifted to the point (2, 3), the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$. Find the original equation of the curve.
2. When the origin is shifted to (- 1, 2) by the translation of axes, find the transformed equation of $x^2 + y^2 + 2x - 4y + 1 = 0$.
3. When the origin is shifted to (- 1, 2) the transformed equation of the curve is $2x^2 + y^2 - 4x + 4y = 0$. Find the original equation.
4. When the origin is shifted to (- 1, 2) the transformed equation of the curve is $x^2 + 2y^2 + 16 = 0$. Find the original equation.
5. When the origin is shifted to (3, 4) the transformed equation of the curve is $x^2 + y^2 = 4$. Find the original equation
6. When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of the curve $3x^2 + 10xy + 3y^2 = 9$.

7. When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of the curve $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$.
8. When the axes are rotated through an angle α , find the transformed equation of the curve $x \cos \alpha + y \sin \alpha = p$
9. When the axes are rotated through an angle 45° , the transformed equation of the curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.
10. Show that the axes are to be rotated through an angle $\frac{1}{2} \tan^{-1} \left(\frac{2h}{a-b} \right)$ so as to remove the xy term from the equation $ax^2 + 2hxy + by^2 = 0$, if $a \neq b$ and through the angle $\frac{\pi}{4}$, if $a = b$.

Straight Lines

1. Find the value of k if the angle between the straight lines $kx + y + 9 = 0$ and $3x - y + 4 = 0$ is $\frac{\pi}{4}$.
2. Find the value of k if the angle between the straight lines $4x - y + 7 = 0$ and $kx - 5y - 9 = 0$ is 45° .
3. Find the value of k , if the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent.
4. Find the value of p , if the lines $3x + 4y = 5$, $2x + 3y = a$ and $px + 4y = 6$ are concurrent.
5. If the straight lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent, then prove that $a^3 + b^3 + c^3 = 3abc$.
6. Show that the lines $2x + 3y - 3 = 0$, $3x + 2y - 2 = 0$ and $2x - 3y - 23 = 0$ are concurrent and find point of concurrent.
7. If $3a + 2b + 4c = 0$, then show that the equation $ax + by + c = 0$ represents a family of concurrent straight lines and find the point of concurrency.
8. Transform the equation $\sqrt{3}x + y = 4$ into (i) slope - intercept form (ii) intercept form and (iii) normal form
9. Transform the equation $4x - 3y + 12 = 0$ into (i) slope - intercept form (ii) intercept form and (iii) normal form
10. Transform the equation $\frac{x}{a} + \frac{y}{b} = 1$ into the normal form when $a > 0$, $b > 0$. If the perpendicular distance of straight line from the origin is p , deduce that $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$.
11. A straight line through $Q(\sqrt{3}, 2)$ makes an angle with the positive direction of X - axis. If the straight line $\sqrt{3}x - 4y + 8 = 0$ at P , find distance PQ .
12. A straight line through $Q(2, 3)$ makes an angle with the positive direction of X - axis. If the straight line $x + y - 7 = 0$ at P , find distance PQ .
13. A straight line with slope 1 passes through $Q(-3, 5)$ and meets the straight line $x + y - 6 = 0$ at P . find the distance PQ .
14. Find the points on the line $3x - 4y - 1 = 0$ which are at a distance 5 units from the point $(3, 2)$.
15. Find the point on the line $3x + y + 4 = 0$ which is equidistance from the points $(-5, 6)$ and $(3, 2)$.
16. Find the equations of the straight lines passing through the point $(-3, 2)$ and making an angle of 45° with the straight line $3x - y + 4 = 0$.

17. Find the equation of the straight line making non – zero equal intercepts on the coordinate axes passing through the point of intersection of the lines $2x - 5y + 1 = 0$ and $x - 3y - 4 = 0$.
18. Find the equation of the straight line parallel to the line $3x + 4y = 7$ and passing through the point of intersection of the lines $x - 2y - 3 = 0$ and $x + 3y - 6 = 0$.
19. Find the equation of the straight line perpendicular to the line $2x + 3y - 1 = 0$ and passing through the point of intersection of the lines $x + 3y - 1 = 0$ and $x - 2y + 4 = 0$.
20. Find the equation of the straight line perpendicular to the line $3x + 4y + 6 = 0$ and making an intercept – 4 on the X – axis.
21. A triangle of area 24 sq. Units is formed by a straight line and the coordinate axes in the first quadrant. Find the equation of that straight line if it passes through (3, 4).

Continuity

1. Check the continuity of f given by $f(x) = \begin{cases} \frac{x^2 - 9}{x^2 - 2x - 3} & \text{if } 0 < x < 5, x \neq 3 \\ 1.5 & \text{if } x = 3 \end{cases}$ at the point $x = 3$
2. Show that $f(x) = \begin{cases} \frac{\cos ax - \cos bx}{x^2}, & \text{if } x \neq 0 \\ \frac{1}{2}(b^2 - a^2), & \text{if } x = 0 \end{cases}$ where a, b are real constants is continuous at $x = 0$.
3. If f is given by $f(x) = \begin{cases} k^2x - k; & \text{if } x \geq 1 \\ 2; & \text{if } x < 1 \end{cases}$ is continuous function on \mathbb{R} , then find the value of k .
4. Is f defined by $f(x) = \begin{cases} \frac{\sin 2x}{x}; & \text{if } x \neq 0 \\ 1; & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$?
5. Find real constants a, b so that the function f is given by $f(x) = \begin{cases} \sin x; & \text{if } x \leq 0 \\ x^2 + a; & \text{if } 0 < x < 1 \\ bx + 3; & \text{if } 1 \leq x \leq 3 \\ -3; & \text{if } x > 3 \end{cases}$ is continuous on \mathbb{R}
6. Check the continuity of the following function at 2
 $f(x) = \begin{cases} \frac{1}{2}(x^2 - 4); & \text{if } 0 < x < 2 \\ 0; & \text{if } x = 2 \\ 2 - 8x^{-3}; & \text{if } x > 2 \end{cases}$
7. Is the function f defined by $f(x) = \begin{cases} x^2; & \text{if } x \leq 1 \\ x; & \text{if } x > 1 \end{cases}$ is continuous on \mathbb{R} ?
8. Show that f is given by $f(x) = \frac{x - |x|}{x}$ is continuous on $\mathbb{R} - \{0\}$.

Differentiaon

1. If $x^y = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$
2. If $y = x^y$, then show that $\frac{dy}{dx} = \frac{y^2}{x(1-\log y)} = \frac{y^2}{x(1-y \log x)}$
3. If $\sin y = x \sin (a + y)$, then show that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$
4. If $\sin x^{2/3} + y^{2/3} = a^{2/3}$, then show that $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$.
5. If $f(x) = a(\cos t + t \sin t)$, $y = a(\sin t - t \cos t)$, then find $\frac{dy}{dx}$.
6. If $f(x) = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $g(x) = \tan^{-1} x$, then differentiate $f(x)$ with respect to $g(x)$
7. Find the derivative of the following functions from the first principles with respect to x .
(i) $\cos^2 x$ (ii) $\sqrt{x+1}$ (iii) $\sec 3x$ (iv) $\tan 2x$ (v) $\cos ax$ (vi) $x \sin x$
(vii) $\cot x$ (viii) $ax^2 + bx + c$ (ix) a^x 4(x) $\log x$ (xi) x^3 (xii) $\sin 2x$
8. If $x = 3 \cos t - 2 \cos^3 t$, $y = 3 \sin t - 2 \sin^3 t$, then find $\frac{dy}{dx}$.
9. If $y = a \cos x + (b + 2x) \sin x$, then show that $y'' + y = 4 \cos x$.
10. If $x = a(t - \sin t)$, $y = a(1 + \cos t)$, then find $\frac{d^2y}{dx^2}$.
11. If $y = \log(4x^2 - 9)$, then find y'' .
12. If $ay^4 = (x + b)^5$, then show that $5y y'' = (y')^2$
13. If $y^x = x^{\sin y}$, then find $\frac{dy}{dx}$.
14. Find the derivative of $\tan^{-1}\left[\frac{3a^2x - x^3}{a(a^2 - 3x^2)}\right]$
15. Show that $f(x) = |x|$ is differentiable at any $x \neq 0$ and is not differentiable at $x = 0$.

Tangents and Normals

1. Find the equations of the tangent and normal to the curve $y = x^2 - 4x + 2$ at $(4, 2)$.
2. Find the equations of the tangent and normal to the curve $xy = 10$ at $(2, 5)$.
3. Find the equations of the tangent and normal to the curve $y = 5x^4$ at $(1, 5)$.
4. Find the equations of the tangent and normal to the curve $y = 2e^{-x/3}$ at the point where the curve meets the Y-axis.
5. Find the equations of the tangent and normal to the curve $y = x^3 + 4x^2$ at the point $(-1, 3)$.
6. Find the equations of the tangent and normal to the curve $y^4 = ax^3$ at the point (a, a) .
7. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.
8. Find the slope of the normal to the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$ at $\theta = \frac{\pi}{4}$.
9. Find the angle between the curves $x + y + 2 = 0$, $x^2 + y^2 - 10y = 0$
10. Find the point at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$.
11. Show that the length of subnormal at any point on the curve $y^2 = 4ax$ is a constant.
12. Show that the length of sub tangent at any point on the curve $y = a^x$ ($a > 0$) is a constant.
13. Show that the length of sub normal at any point on the curve $xy = a^2$ varies as the cube of the ordinate of the point.

14. Show that at any point (x, y) on the curve $y = b e^{x/a}$, the length of sub tangent is a constant and the length of subnormal is $\frac{y^2}{a}$.
15. Show that the tangent at any point θ on the curve $x = C \sec \theta$, $y = C \tan \theta$ is $y \sin \theta = x - C \cos \theta$.
16. Find the lengths of the sub tangent and subnormal at a point on the curve $y = b \sin \left(\frac{x}{a}\right)$.
17. Find the lengths of the normal and subnormal at a point on the curve $y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$.
18. Find the value of k so that the length of the subnormal of at any point on the curve $y = a^{1-k} x^k$ is a constant.

Rate Measure

1. A particle is moving along a line according to $s = f(t) = 4t^3 - 3t^2 + 5t - 1$ where s is measured in meters and t is measured in seconds. Find the velocity and acceleration at time t . At what time the acceleration is zero.
2. A particle is moving in a straight line so that after t seconds its distance is s (in centimeters) from a fixed point on the line is given by $s = f(t) = 8t + 3t^3$ find (i) the velocity at time $t = 2$ seconds (ii) the initial velocity (iii) the acceleration at $t = 2$ seconds.
3. The distance - time formula for the motion of a particle along a straight line $S = t^3 - 9t^2 + 24t - 18$, then find when and where the velocity is zero.
4. Find the average rate of change of $s = f(t) = 2t^3 + 3$ between $t = 2$ and $t = 4$.
5. A point P is moving on the curve $y = 2x^2$. The coordinates of P is increasing at the rate of 4 units per second. Find the rate at which the y -coordinate is increasing when the point is at $(2, 8)$.
6. The volume of a cube is increasing at rate of $9 \text{ cm}^3 / \text{sec}$. How fast is the surface area increasing when the length of the edge is 10 cm ?
7. The volume of a cube is increasing at rate of $8 \text{ cm}^3 / \text{sec}$. How fast is the surface area increasing when the length of the edge is 12 cm ?
8. A container in the shape of an inverted cone has height 12 cm and radius 6 cm at the top. If it is filled with water at the rate of 8 centimeter cube per second. What is the rate of change in the height of the water level when the tank is filled 8 cm ?
9. A container in the shape of an inverted cone has height 4 m and radius 6 m at the top. If it is filled with water at the rate of 2 meter cube per second. What is the rate of change in the height of the water level when the tank is filled 4 m ?
10. The total cost $C(x)$ in rupees associated with production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 500$. Find the marginal cost when 3 units are produced.
11. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of balloon increases when the radius is 15 cm .
12. A stone is dropped into a quite lake and ripples move in circles at the speed of 5 cm/sec . At the instant when the radius of circular ripple is 8 cm , how fast is the enclosed area increases?
13. Suppose we have a rectangular aquarium with dimensions of length 8 m , width 4 m and height 3 m . Suppose we are filling the tank with water at the rate of $0.4 \text{ m}^3 / \text{sec}$. How fast is the height of the water changing when the water level is 2.5 cm ?
14. The radius of a circle is increasing at the rate of 0.07 cm/sec . what is the rate of increase of its circumference.