PRACTICE PAPER –I (2021)

TOTAL MARKS: 75
TIME: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. On what domain of the function $f(x) = x^2 - 2x$ and $g(x) = -x + 6$ are equal?
2. Find the domain and range of the function $\frac{2x^2 - 5x + 7}{(x-1)(x-2)(x-3)}$.
3. If $A = \begin{pmatrix} 2 & 3 & -1 \\ 7 & 8 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -4 & -1 \end{pmatrix}$ then find A + B
4. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix, then find x.
5. Find unit vector in the direction of vector $\bar{a} = 2\bar{i} + 3\bar{j} + \bar{k}$.
6. Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$
7. If $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}$ and $\bar{b} = \bar{i} - 3\bar{j} - 5\bar{k}$, then find $|\bar{a} \times \bar{b}|$
8. If $\sin\alpha = \frac{1}{\sqrt{10}}$, $\sin\beta = \frac{1}{\sqrt{5}}$ and α, β are acute, show that $\alpha + \beta = \frac{\pi}{4}$.
9. If $\cos A = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$, find the value of cos 2A.
10. If $\cos hx = \frac{5}{2}$, find the values of (i) cosh 2x and (ii) sinh 2x.

II. Short answer type questions $5 \times 4 = 20$

11. Find the adjoint and inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$
12. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, show that for all the positive integers n $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$
13. **a**, **b**, **c** are non-coplanar vectors. Prove that the following points are coplanar.
    6**a** + 2**b** – **c**, 2**a** – **b** + 3 **c**, – **a** + 2**b** – 4**c** and – 12**a** – **b** – 3**c**
14. Prove that the angle θ between any two diagonals of a cube is given by $\cos\theta = \frac{1}{3}$
15. Find a vector magnitude 3 and perpendicular to both the vectors $\bar{b} = 2\bar{i} - 2\bar{j} + \bar{k}$ and $\bar{c} = 2\bar{i} + 2\bar{j} + 3\bar{k}$
16. If sec (θ + α) + sec (θ – α) = 2 sec θ and cos α ≠ 1, then show that $\cos\theta = \pm\sqrt{2}\cos\frac{\alpha}{2}$.
17. If $A + B = \frac{\pi}{4}$, then prove that (cot A – 1) (cot B – 1) = 2.
18. Prove that $\frac{\cosh x}{1-\tanh x} + \frac{\sin hx}{1-\coth x} = \sinh x + \cosh x$.
19. If a = (b – c) sec θ, then prove that $\tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2}$
20. Express $\frac{a\cos A + b\cos B + c\cos c}{a + b + c}$ in terms of R and r.

III.Long answer type questions **5 × 7 = 35**

21. If the function f is defined by $f(x) = \begin{cases} 3x - 2, & x > 3 \\ x^2 - 2, & -2 \leq x \leq 2 \\ 2x + 1, & x < -3 \end{cases}$, then find the values if they exist of $f(4), f(2.5), f(-4), f(0)$ and $f(-7)$.
22. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_2 \end{bmatrix}$ is a non-singular matrix, then prove that A is invertible and then show that $A^{-1} = \frac{\text{adj } A}{\det A}$.
23. By using **Cramer's rule**, solve the following system of equations. $x - y + 3z = 5$, $4x + 2y - z = 0$ and $-x + 3y + z = 5$.
24. Solve the following equation by using **Matrix inversion method** $2x - y + 3z = 8$, $-x + 2y + z = 4$, $3x + y - 4z = 0$.
25. Let OABC be a parallelogram and D is the midpoint of OA, prove that the segment CD trisects the diagonal OB and it is trisected by the diagonal OB.
26. A non-zero vector **a** is parallel to the line of intersection of the plane determined by the vectors **i**, **i** + **j**, and the plane determined by the vectors **i** – **j**, **i** + **k**. Find the angle between **a** and the vector **i** – 2**j** + 2**k**.
27. For any two vectors **a** and **b**, prove that $(1 + |\mathbf{a}|^2)(1 + |\mathbf{b}|^2) = |1 - \mathbf{a}.\mathbf{b}|^2 + |\mathbf{a} + \mathbf{b} + \mathbf{a} \times \mathbf{b}|^2$
28. If $A + B + C = \frac{3\pi}{2}$, prove that $\cos 2A + \cos 2B + \cos 2C = 1 - 4\sin A \sin B \sin C$.
29. If $r_1 = 2$, $r_2 = 3$, $r_3 = 6$ and $r = 1$, then prove that $a = 3$, $b = 4$ and $c = 5$.
30. Show that $\frac{ab - r_1 r_2}{r_3} = \frac{bc - r_2 r_3}{r_1} = \frac{ca - r_3 r_1}{r_2}$

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