PRACTICE PAPER –II (2021)

TOTAL MARKS: 75
TIME: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. If $A = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$ and $f : A \longrightarrow B$ is a surjection defined by $f(x) = \cos x$, then find B.
2. Find the range of the function $f(x) = \log|4 - x^2|$.
3. Find the additive inverse of A, where $A = \begin{bmatrix} i & 0 & 1 \\ 0 & -i & 2 \\ -1 & 1 & 5 \end{bmatrix}$.
4. Find the minors of – 1 and 3 in the matrix $\begin{bmatrix} 2 & -1 & 4 \\ 0 & -2 & 5 \\ -3 & 1 & 3 \end{bmatrix}$.
5. Show that the points whose position vectors are $-2\mathbf{a} + 3\mathbf{b} + 5\,\mathbf{c}$, $\mathbf{a} + 2\mathbf{b} + 3\,\mathbf{c}$, $7\,\mathbf{a} - \mathbf{c}$ are collinear when **a**, **b**, **c** are non-coplanar vectors.
6. Using the vector equation of the line passing through the two points, prove that the points whose position vectors are **a**, **b** and (3 **a** – 2**b**) are collinear.
7. If $\bar{a} = 2\bar{i} - 3\bar{j} + 5\bar{k}$ and $\bar{b} = -\bar{i} + 4\bar{j} + 2\bar{k}$, then find $\bar{a} \times \bar{b}$ and unit vector perpendicular to both $\bar{a}$ and $\bar{b}$.
8. Find the period of the function $f(x) = \tan(x + 4x + 9x + \ldots + n^2x)$ (n is any positive integer)
9. Find the minimum and maximum values of the function $f(x) = 3\cos x + 4\sin x$.
10. For any $x \in R$, prove that $\cosh^4 x - \sinh^4 x = \cosh 2x$.

II. Short answer type questions $5 \times 4 = 20$

11. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then show that $A^{-1} = A^3$.
12. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I = 0$.
13. Show that the line joining the pair points $6\mathbf{a} - 4\mathbf{b} + 4\mathbf{c}$, $-4\mathbf{c}$ and the line joining the pair of points $-\mathbf{a} - 2\mathbf{b} - 3\,\mathbf{c}$, $\mathbf{a} + 2\mathbf{b} - 5\mathbf{c}$ intersects at the point $-4\mathbf{c}$, when **a**, **b**, **c** are non-coplanar vectors.
14. Find the area of the triangle whose vertices are A (1, 2, 3), B (2, 3, 1) and C (3, 1, 2).
15. Let $\bar{a}, \bar{b}$ be vectors satisfying $|\bar{a}| = |\bar{b}| = 5$ and $(\bar{a}, \bar{b}) = 45^0$. Find the area of the triangle having $\bar{a} - 2\bar{b}$, and $3\bar{a} + 2\bar{b}$ as two of its sides.
16. Prove that $\sin\frac{\pi}{5}.\sin\frac{2\pi}{5}.\sin\frac{3\pi}{5}.\sin\frac{4\pi}{5} = \frac{5}{16}$.
17. For any $\alpha \in R$, prove that $\cos^2\left(\alpha - \frac{\pi}{4}\right) + \cos^2\left(\alpha + \frac{\pi}{12}\right) - \cos^2\left(\alpha - \frac{\pi}{12}\right) = \frac{1}{2}$

18. If $u = \log_e\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$ and if $\cos\theta > 0$, then prove that $\cosh u = \sec\theta$.

19. If $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = 3 : 5 : 7$ show that a: b:c = 6 : 5: 4.

20. Show that $\frac{1}{r^2} + \frac{1}{{r_1}^2} + \frac{1}{{r_2}^2} + \frac{1}{{r_3}^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$.

III.Long answer type questions **$5 \times 7 = 35$**

21. Determine whether the function f : A ⟶ B is defined by $f(x) = \begin{cases} x, \text{ if } x > 2 \\ 5x - 2, \text{ if } x \leq 2 \end{cases}$ is an injection or surjection or a bijection.

22. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then find $A^3 - 3A^2 - A - 3I$, where I is unit matrix of order 3.

23. By using **Cramer's rule**, solve the following system of equations.
$2x - y + 3z = 9$, $x + y + z = 6$ and $x - y + z = 2$.

24. Solve the following equation by using **Matrix inversion method**
$x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$.

25. Find the vector equation of the plane passing through the points $4\bar{i} - 3\bar{j} - \bar{k}$, $3\bar{i} + 7\bar{j} - 10\bar{k}$ and $2\bar{i} + 5\bar{j} - 7\bar{k}$ and show that the point $\bar{i} + 2\bar{j} - 3\bar{k}$ lies on the plane.

26. If **a**, **b**, **c** are unit vectors such that **a** is perpendicular to the plane of **b**, **c** and the angle between **b** and **c** is $\frac{\pi}{3}$, then find $|\mathbf{a} + \mathbf{b} + \mathbf{c}|$.

27. G is the centroid of the triangle ABC and a, b, c are the lengths of the sides of BC, CA, AB respectively. Prove that $a^2 + b^2 + c^2 = 3(OA^2 + OB^2 + OC^2) - 9(OG)^2$, where O is any point.

28. If none of A, B, A+ B is an integral multiple of π, then prove that
$\frac{1 - \cos A + \cos B - \cos(A+B)}{1 + \cos A - \cos B - \cos(A+B)} = \tan\frac{A}{2}\cot\frac{B}{2}$.

29. Prove that $r + r_3 + r_1 - r_2 = 4R\cos B$.

30. In ΔABC, if AD, BE, CF are the perpendiculars drawn from the vertices A, B, C to the opposite sides, show that (i) $\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{1}{r}$ (ii) $AD.BE.CF = \frac{(abc)^2}{8R^3}$

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