## PRACTICE PAPER –III (2021)

**TOTAL MARKS: 75**
**TIME: 3hrs.**

**I. Very short answer type questions** **$10 \times 2 = 20$**

1. Find the domain of the function y(x), given by the equation $2^x + 2^y = 2$.
2. Find the domain of the real valued function $f(x) = \frac{\sqrt{2+x} + \sqrt{2-x}}{x}$.
3. Define trace of the matrix. Find the trace of A if $A = \begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & -1 & 2 \\ -\frac{1}{2} & 2 & 1 \end{bmatrix}$.
4. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then show that AA' = A'A = I.
5. Is the triangle formed by the vertices $3\bar{i} + 5\bar{j} + 2\bar{k}, 2\bar{i} - 3\bar{j} - 5\bar{k}$ and $-5\bar{i} - 2\bar{j} + 3\bar{k}$ equilateral?
6. Find the vector equation of a plane passing through the points (0, 0, 0), (0, 5, 0) and (2, 0, 1).
7. For what values of λ, the vectors $\bar{i} - \lambda\bar{j} + 2\bar{k}$ and $8\bar{i} + 6\bar{j} - \bar{k}$ are at right angles?
8. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.
9. Express $\frac{\sin 4\theta}{\sin\theta}$ in terms of $\cos^3\theta$ and $\cos\theta$.
10. If sinh x = 5, then show that $x = \log_e(5 + \sqrt{26})$.

**II. Short answer type questions** **$5 \times 4 = 20$**

11. If $A = \begin{bmatrix} 7 & -2 \\ -1 & 2 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -1 \\ 4 & 2 \\ -1 & 0 \end{bmatrix}$, then find AB' and BA'.
12. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then show that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$.
13. If the points whose position vectors are $-2\mathbf{a} + 3\mathbf{b} + 5\mathbf{c}$, $\mathbf{a} + 2\mathbf{b} + 3\mathbf{c}$, $7\mathbf{a} - \mathbf{c}$, are collinear, when **a**, **b**, **c** are non-coplanar vectors.
14. Find the unit vector orthogonal to the vectors $3\bar{i} + 2\bar{j} + 6\bar{k}$ and coplanar with the vectors $2\bar{i} + \bar{j} + \bar{k}$ and $\bar{i} - \bar{j} + \bar{k}$.
15. Let $\bar{a} = 4\bar{i} + 5\bar{j} - \bar{k}$, $\bar{b} = \bar{i} - 4\bar{j} + 5\bar{k}$ and $\bar{c} = 3\bar{i} + \bar{j} - \bar{k}$. Find the vector which is perpendicular to both $\bar{a}$ and $\bar{b}$ whose magnitude is twenty-one times the magnitude of $\bar{c}$.
16. If tan (A + B) = λ tan (A – B), then show that (λ + 1) sin 2B = (λ – 1) sin 2A.
17. If $\cos\alpha = \frac{3}{5}$ and $\cos\beta = \frac{5}{13}$ and α, β are acute angles, then prove thar (a) $\sin^2\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{65}$ and (b) $\cos^2\left(\frac{\alpha+\beta}{2}\right) = \frac{16}{65}$.

18. Prove that for any $x \in R$ $\tanh 3x = \frac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$

19. Prove that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \frac{s^2}{\Delta}$.

20. If $r : R : r_1 = 2 : 5 : 12$, then prove that the triangle is right angle at A.

**III.Long answer type questions** **$5 \times 7 = 35$**

21. Determine whether the function $f: R \longrightarrow R$ defined as $f(x + y) = f(x) + f(y)\ \forall\ x, y \in R$ and $f(1) = 7$ then find $\sum_{i=1}^{n} f(r)$.

22. If $\theta - \emptyset = \frac{\pi}{2}$, then show that $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\emptyset & \cos\emptyset\sin\emptyset \\ \cos\emptyset\sin\emptyset & \sin^2\emptyset \end{bmatrix} = 0$

23. By using **Cramer's rule**, solve the following system of equations.
$2x - y + 8z = 13$, $3x + 4y + 5z = 18$ and $5x - 2y + 7z = 20$.

24. Solve the following equation by using **Matrix inversion method**
$5x - 6y + 4z = 15$, $7x + 4y - 3z = 19$, $2x + y + 6z = 46$.

25. Find the vector equation of the plane passing through the points $2\bar{i} + 4\bar{j} + 2\bar{k}$, $2\bar{i} + 3\bar{j} + 5\bar{k}$ and parallel the vector $3\bar{i} - 2\bar{j} + \bar{k}$. Also find the where this plane meets the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $4\bar{i} - 2\bar{j} + 3\bar{k}$.

26. A line makes an angles $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$ with the diagonals of a cube. Show that
$\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 + \cos^2\theta_4 = \frac{4}{3}$.

27. If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}$, $\bar{b} = \bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} - \bar{j} + \bar{k}$, then compute $\bar{a} \times (\bar{b} \times \bar{c})$ and verify that it is perpendicular to $\bar{a}$.

28. If $A + B + C = 2S$, then prove that
$\cos(S - A) + \cos(S - B) + \cos(S - C) + \cos S = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

29. If $a = 13$, $b = 14$, $c = 15$ then show that $R = \frac{65}{8}$, $r = 4$, $r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.

30. Prove that $a^3\cos(B - C) + b^3\cos(C - A) + c^3\cos(A - B) = 3abc$.

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