PRACTICE PAPER –IV (2021)

TOTAL MARKS: 75
TIME: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. If f : R – {0} R is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f(1/x) = 0$.
2. Find the domain of the real valued function $f(x) = \sqrt{\log_{10}\left(\frac{3-x}{x}\right)}$.
3. If $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ then find AA'. Do A and A' commutate with respect to multiplication of matrices?
4. Find the adjoint and inverse of the matrix $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.
5. Write the direction ratios of the vector $\bar{a} = \bar{i} + \bar{j} - 2\bar{k}$ and hence calculate its direction cosines.
6. OABC is a parallelogram. If **OA** = **a** and **OC** = **c**, find the vector equation of the side BC.
7. If $\bar{a} = 6\bar{i} + 2\bar{j} + 3\bar{k}$. $\bar{b} = 2\bar{i} - 9\bar{j} + 6\bar{k}$, then find $\bar{a} \cdot \bar{b}$ and the angle between $\bar{a}, \bar{b}$.
8. Sketch the graph of the function $f(x) = \sin 2x$ in the interval $(0, \pi)$.
9. Prove that $\cos^2\theta + \cos^2(2\pi/3 + \theta) + \cos^2(2\pi/3 - \theta) = 3/2$.
10. If $\cosh x = \sec\theta$, then prove that $\tanh^2(x/2) = \tan^2(\theta/2)$.

II. Short answer type questions $5 \times 4 = 20$

11. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then show that $(aI + bE)^3 = a^3I + 3a^2b\,E$.
12. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, then show that $A^{-1} = A'$.
13. In the two-dimensional plane, prove by using vector methods, the equation of the whose intercepts on the axes are a and b is $\frac{x}{a} + \frac{y}{b} = 1$.
14. Find the vector having magnitude $\sqrt{6}$ units and perpendicular to both $2\bar{i} - \bar{k}$ and $3\bar{i} - \bar{j} - \bar{k}$.
15. Show that the points (5, –1, 1), (7, –4, 7), (1, –6, 10) and (–1, –3, 4) are the vertices of a Rhombus.
16. If $\cos x + \cos y = \frac{4}{5}$ and $\cos x - \cos y = \frac{2}{7}$, find the value ofn14 $\tan\left(\frac{x-y}{2}\right) + 5\cot\left(\frac{x+y}{2}\right)$.
17. Prove that $\tan\alpha = \frac{\sin 2\alpha}{1+\cos 2\alpha}$ and hence, deduce the values of $\tan 15^0$ and $\tan 22\frac{1}{2}^0$
18. Prove that $\frac{\tan hx}{\operatorname{sech} x - 1} + \frac{\tan hx}{\operatorname{sech} x + 1} = -2\operatorname{cosech} x$, for $x \neq 0$.

19. In Δ ABC, if $a \cos A = b \cos B$, prove that the triangle is either isosceles or right angled.

20. Show that $r_1 . r_2 . r_3 = r^3 \cot^2\frac{A}{2} \cot^2\frac{B}{2} \cot^2\frac{C}{2}$.

III.Long answer type questions **5 × 7 = 35**

21. If the function f is defined by $f(x) = \begin{cases} x + 2, & x > 1 \\ 2, & -1 \leq x \leq 1 \\ x - 1, & -3 < x < -1 \end{cases}$, then find the values of (i) f(3) (ii) f(0) (iii) f(– 1.5) (iv) f(2) + f(–2) (v) f(–5)

22. Solve $3x + 4y + 5z = 18$, $2x - y + 8z = 13$ and $5x - 2y + 7z = 20$, by using matrix inversion method.

23. By using **Cramer's rule**, solve the following system of equations.
$x + y + z = 9$, $2x + 5y + 7z = 52$ and $2x + y - z = 0$.

24. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \frac{1}{2}\begin{bmatrix} b+c & c-a & b-a \\ c-b & c+a & a-b \\ b-c & a-c & a+b \end{bmatrix}$, then show that ABA^{-1} is a diagonal matrix.

25. Find the point of intersection of the line $\bar{r} = 2\bar{a} + \bar{b} + t(\bar{b} - \bar{c})$ and the plane $\bar{r} = \bar{a} + x(\bar{b} + \bar{c}) + y(\bar{a} + 2\bar{b} - \bar{c})$ where $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors.

26. If $\bar{a} = 7\bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + 8\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$, then compute $\bar{a} \times \bar{b}$, $\bar{a} \times \bar{c}$ and $\bar{a} \times (\bar{b} + \bar{c})$. Verify whether cross product is distributive over the vector addition.

27. If **a**, **b**, **c** are unit vectors such that **a** is perpendicular to the plane of **b**, **c** and angle between **b** and **c** is $\frac{\pi}{3}$, then find $|\bar{a} + \bar{b} + \bar{c}|$.

28. If A, B, C are angles in a triangle, then prove that
$\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = 1 + 4\sin\frac{\pi - A}{4} \sin\frac{\pi - B}{4} \sin\frac{\pi - C}{4}$

29. Show that $\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ba} = \frac{1}{r} - \frac{1}{2R}$.

30. If $\cos A + \cos B + \cos C = \frac{3}{2}$, then show that the triangle is equilateral.

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