

**I. Very short answer type questions**

**10 × 2 = 20**

1. Find the equation of the straight line passing through the point (2, 3) and making non-zero intercepts on the axes of coordinates whose sum is zero.
2. Find the value of 'k', if the lines  $2x - 3y + k = 0$ ,  $3x - 4y - 13 = 0$  and  $8x - 11y - 33 = 0$  are concurrent.
3. Find the ratio in which YZ - plane divides the line joining A (2, 4, 5) and B (3, 5, - 4). Also find the point of intersection.
4. Find the direction cosines of the normal to the plane  $x + 2y + 2z - 4 = 0$ .
5. Find  $\lim_{x \rightarrow 0} \left( \frac{\sqrt{1+x}-1}{x} \right)$ .
6. Compute  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+6}}{2x^2-1}$ .
7. If  $f(x) = 1 + x + x^2 + \dots + x^{100}$ , then find  $f'(1)$ .
8. If  $f(x) = \log (\sec x + \tan x)$ , then find  $f'(x)$ .
9. Find  $dy$  and  $\Delta y$  of  $y = f(x) = x^2 + x$  at  $x = 10$  when  $\Delta x = 0.1$ .
10. Find the slope of the tangent to the curve  $y = 5x^2$  at  $(-1, 5)$ .

**II. Short answer type questions**

**5 × 4 = 20**

11. Find the equation of the locus of P such that  $PA^2 + PB^2 = 2c^2$ , where  $A = (a, 0)$ ,  $B = (-a, 0)$  and  $0 < |a| < |c|$
12. If the distance from P to the points (2, 3) and (2, - 3) are in the ratio 2:3, then find the equation of locus of P.
13. When the origin is shifted to the point (2, 3), the transformed equation of the curve is  $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$ . Find the original equation of the curve.
14. When the axes are rotated through an angle  $\frac{\pi}{6}$ , find the transformed equation of  $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ .
15. Find the equations of the straight lines passing through the point (-3, 2) and making an angle  $45^\circ$  with the straight line  $3x - y + 4 = 0$ .
16. Find the ratio in which the point P (5, 4, - 6) divides the line segment joining the points A (3, 2, - 4) and B (9, 8, - 10). Also, find the harmonic conjugate of P.
17. Compute  $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x}$ .

18. Differentiate  $f(x)$  with respect to  $g(x)$  for the following functions.

$$f(x) = \tan^{-1} \left( \frac{2x}{1-x^2} \right) \text{ and } f(x) = \sin^{-1} \left( \frac{2x}{1+x^2} \right)$$

19. Show that the length of the sub normal at any point on the curve  $y^2 = 4ax$  is a constant.

20. Find the equations of the tangent and normal to the curve  $xy = 10$  at  $(2, 5)$ .

### III. Long answer type questions

5 × 7 = 35

21. Find the circum centre of the triangle whose sides are  $3x - y - 5 = 0$ ,  $x + 2y - 4 = 0$  and  $5x + 3y + 1 = 0$ .

22. Find the orthocentre of the triangle whose vertices are  $(5, -2)$ ,  $(-1, 2)$  and  $(1, 4)$ .

23. Show that the product of the perpendicular distances from a point  $(\alpha, \beta)$  to the pair of straight lines  $ax^2 + 2hxy + by^2 = 0$  is  $\frac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(a-b)^2 + 4h^2}}$ .

24. Show that the lines joining the origin to the points of intersection of the curve  $x^2 - xy + y^2 + 3x + 3y - 2 = 0$  and the straight-line  $x - y - \sqrt{2} = 0$  are mutually perpendicular.

25. Find the angle between the lines whose direction cosines satisfy the equations  $l + m + n = 0$  and  $l^2 + m^2 - n^2 = 0$ .

26. Find the derivative of the function  $f(x) = \cos^{-1} \left( \frac{b + a \cos x}{a + b \cos x} \right)$  ( $a > 0$ ,  $b > 0$ ).

27. If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , then show that  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ .

28. If the tangent at any point on the curve  $x^{2/3} + y^{2/3} = a^{2/3}$  intersects the coordinate axes in A and B, then show that the length AB is constant.

29. Find the angle between the curves  $2y^2 - 9x = 0$  and  $3x^2 + 4y = 0$  (in 4<sup>th</sup> quadrant).

30. A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 20 feet. find the maximum area.

