PRACTICE PAPER – II (2021)

TOTAL MARKS: 75
Time: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. If the area of the triangle formed by formed by the straight lines $x = 0$, $y = 0$ and $3x + 4y = a$ $(a > 0)$ is 6, find the value of a.
2. Find the ratio in which the straight line $2x + 3y - 20 = 0$ divides the join of the points (2, 3) and (2, 10).
3. Find the fourth vertex of the parallelogram whose consecutive vertices are (2, 4, – 1), (3, 6, – 1) and (4, 5, 1).
4. Find the equation of the plane if the foot of the perpendicular from origin to the point is (1, 3, – 5).
5. Compute $\lim_{x\to 0} \frac{e^x - 1}{x}$
6. Evaluate $\lim_{x\to \infty} \frac{3x^5 - 1}{4x^2 + 1}$.
7. Find the derivative of the function $f(x) = \mathrm{Tan}^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$
8. If $y = \mathrm{cosec}^{-1}(e^{2x+1})$, find $\frac{dy}{dx}$.
9. If the increase in the side of a square is 2% then find the approximate percentage of increase in its area.
10. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$, $x \neq 0$ at $x = 10$.

II. Short answer type questions $5 \times 4 = 20$

11. Find the equation of the locus of P, if the ratio of the distance from P to A (5, – 4) and B (7, 6) is 2:3.
12. Find the equation of the locus of P, if the line segment joining (2, 3) and (– 1, 5) subtend a right angle at P.
13. When the origin is shifted to the point (– 1, 2) by the translation of axes, find the transformed equation of the curve $2x^2 + y^2 - 4x + 4y = 0$.
14. When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of $3x^2 + 10xy + 3y^2 = 9$.
15. A straight line through Q (2, 3) makes an angle $\frac{3\pi}{4}$ with the negative direction of X – axis. If the straight line intersects the line $x + y - 7 = 0$ at P. Find the distance PQ.
16. Show that the points A (3, 2, – 4) and A (5, 4, – 6) and C A (9, 8, – 10) are collinear and find the ratio in which B divides AC.

17. Compute $\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx}$, $n \neq 0$.

18. If $x = a\,[\cos t + \log \tan \left(\frac{t}{2}\right)]$, $y = a \sin t$, then find $\frac{dy}{dx}$.

19. Show that the tangent at any point θ on the curve $x = c \sec \theta$, $y = c \tan \theta$ is $y \sin \theta = x - \cos \theta$.

20. Show that the curves $6x^2 - 5x + 2y = 0$ and $4x^2 + 8y^2 = 3$ touch each other at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

III. Long answer type questions **$5 \times 7 = 35$**

21. Find the circum centre of the triangle whose sides are $x + y + 2 = 0$, $5x - y - 2 = 0$ and $x - 2y + 5 = 0$.

22. If p and q are the lengths of the perpendiculars from the origin to the straight lines $x \sec \alpha + y \operatorname{cosec} \alpha = a$ and $x \cos \alpha - y \sin \alpha = a \cos 2\alpha$, prove that $4p^2 + q^2 = a^2$.

23. Show that the lines represented by $(lx + my)^2 - 3(mx - ly)^2 = 0$ and $lx + my + n = 0$ form an equilateral triangle with area $\frac{n^2}{\sqrt{3}(l^2 + m^2)}$.

24. Find the values of k, if the lines joining the origin to the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.

25. Show that lines whose direction cosines satisfy the equations $l + m + n = 0$ and $2mn + 3nl - 5lm = 0$ are perpendicular each other.

26. If $y = \mathrm{Tan}^{-1}\left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right]$ for $0 < |x| < 1$, find $\frac{dy}{dx}$.

27. If $x^{2/3} + y^{2/3} = a^{2/3}$, then show that $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$.

28. Show that the tangent at any point $P(x_1, y_1)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $yy_1^{-\frac{1}{2}} + xx^{-\frac{1}{2}} = a^{\frac{1}{2}}$.

29. Show that the curves $y^2 = 4(x + 1)$ and $y^2 = 36(9 - x)$ intersects orthogonally.

30. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that the cone.

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