

I. Very short answer type questions

10 × 2 = 20

1. Find the angle between the straight-line $y = \sqrt{3}x - 4$ makes with the Y - axis.
2. Find the area of the triangle formed by the straight-line $x - 4y + 2 = 0$ with the coordinate axes.
3. Find x, if the distance between $(5, -1, 7)$ and $(x, 5, 1)$ is 9 units.
4. Reduce the equation $x + 2y - 3z - 6 = 0$ of the plane in to the normal form.
5. Evaluate $\lim_{x \rightarrow 1} \frac{\log_e x}{x-1}$
6. Compute $\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x})$.
7. If $y = x^2 e^x \sin x$, then find $\frac{dy}{dx}$.
8. Find the derivative of the function $x = \tanh^2 y$.
9. The side of a square is increased from 3cm to 3.01cm. Find the approximate increasing in the area of the square.
10. Find the length of subtangent and subnormal at a point on the curve $y = b \sin \frac{x}{a}$.

II. Short answer type questions

5 × 4 = 20

11. Find the equation of the locus of P, if the ratio of the distance from P to A $(5, -4)$ and B $(7, 6)$ is 2:3.
12. Find the equation of locus of a point, sum of whose distance from $(0, 2)$ and $(0, -2)$ is 6
13. When the origin is shifted to the point $(-1, 2)$ by the translation of axes, find the transformed equation of the curve $x^2 + y^2 + 2x - 4y + 1 = 0$.
14. Show that the axes are to be rotated through an angle of $\frac{1}{2} \tan^{-1} \left(\frac{2h}{a-b} \right)$ so as to remove the 'xy' term from the equation $ax^2 + 2hxy + by^2 = 0$, if $a \neq b$ and through an angle $\frac{\pi}{4}$ if $a = b$.
15. Find the values of k, if the angle between the straight lines $kx + y + 9 = 0$ and $3x - y + 4 = 0$ is $\frac{\pi}{4}$.
16. A $(3, 2, 0)$, B $(5, 3, 2)$, C $(-9, 6, -3)$ are vertices of a triangle. AD is the bisector of $\angle BAC$ meets BC at D. find the coordinates of D.
17. Compute $\lim_{x \rightarrow \infty} \frac{x^2 - \sin x}{x^2 - 2}$.
18. Find the derivative of the function $f(x) = \sec 3x$, from the first principle of derivative.
19. If the slope of the tangent to the curve $y = x \log x$ at a point on it is $\frac{3}{2}$, then find the equation of the tangent and normal at that point.

20. Show that the square of the length of subtangent at any point on the curve $by^2 = (x + a)^3$ ($b \neq 0$) varies with the subnormal at that point.

III. Long answer type questions

5 × 7 = 35

21. Find the ortho centre of the triangle formed by the lines $x + 2y = 0$, $4x + 3y - 5 = 0$ and $3x + y = 0$.
22. Find the equations of the straight lines passing through the point of intersection of the lines $3x + 2y + 4 = 0$, $2x + 5y = 1$ and whose distance from $(2, -1)$ is 2.
23. Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is $\frac{n^2\sqrt{h^2-ab}}{|am^2-2hlm+bl^2|}$.
24. Write down the equation of the pair of straight lines joining the origin to the points of intersection of straight line $6x - y + 8 = 0$ with the pair of straight lines $3x^2 + 4xy - 4y^2 - 11x + 2y + 6 = 0$.
Show that the lines so obtained make equal angles with the coordinate axes.
25. If a ray makes an angles α , β , γ and δ with the four diagonals of a cube, then find $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta$.
26. Find the derivative of the function $f(x) = \text{Tan}^{-1} \left(\frac{\cos x}{1 + \cos x} \right)$.
27. If $x^{\log y} = \log x$, then prove that $\frac{dy}{dx} = \frac{y}{x} \left[\frac{1 - \log x \log y}{\log^2 x} \right]$.
28. At a point P (x_1, y_1) on the curve $x^3 + y^3 = 3axy$, show that the tangent is $(x_1^2 - ay_1)x + (y_1^2 - ax_1)y = ax_1y_1$
29. Find the angle between the curves $xy = 2$ and $x^2 + 4y = 0$.
30. From a rectangular sheet of dimensions $30\text{cm} \times 80\text{cm}$ four equal squares of side 'x' cm are removed at the corners and the sides or then turned up so as to form the open rectangular box.
Find the value of 'x' so that the volume of the box is the greatest.

