PRACTICE PAPER – IV (2021)

TOTAL MARKS: 75
Time: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. Prove that the points (1, 11), (2, 15) and (–3, –5) are collinear and find the equation of the straight line containing them.
2. Find the angle between the lines $2x + y + 4 = 0$ and $y - 3x = 7$.
3. If (3, 2, –1), (4, 1, 1) and (6, 2, 5) are three vertices and (4, 2, 2) is the centroid of a tetrahedron, find the fourth vertex.
4. Find the angle between the planes $x + 2y + 2z - 5 = 0$ and $3x + 3y + 2z - 8 = 0$.
5. Compute $\log_{x\to 0} \frac{\sin ax}{\sin bx}$ $(b \neq 0, a \neq b)$
6. Find $\lim_{x\to\infty} \frac{8|x|+3x}{3|x|-2x}$.
7. Find the derivative of the function $f(x) = \log_7(\log_e x)$ $(x > 0)$.
8. If $y = \left(\text{Cot}^{-1}(x^3)\right)^2$ find $\frac{dy}{dx}$.
9. Find the approximate value of $\sqrt[3]{999}$.
10. Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

II. Short answer type questions $5 \times 4 = 20$

11. Find the equation of the locus of P, if the distance of P from A (3, 0) is twice the distance P from B (– 3, 0).
12. The ends of the hypotenuse of right-angled triangle are (0, 6) and (6, 0). Find the equation of locus of its third vertex.
13. When the origin is shifted to (3, 4) by the translation of axes, then find the transformed equation of the curve $2x^2 + 4xy + 5y^2 = 0$.
14. When the axes are rotated through an angle α. Find the transformed equation of $x\cos\alpha + y\sin\alpha = p$
15. Transform the equation $3x + 4y + 12 = 0$ into (i) slope-intercept form (ii) intercept form and (iii) Normal form.
16. Show that the points O (0, 0, 0) A (2, – 3, 3) B (– 2, 3, – 3) are collinear. Find the ratio in which each point divides the line segment joining the one other.
17. Compute $\lim_{x\to\infty} \frac{6x^2 - \cos 3x}{x^2 + 5}$.
18. Find the derivative of the function $f(x) = \sqrt{x+1}$, from the first principle of derivative.

19. Find the tangent and normal to the curve $y = 2\,e^{\frac{-x}{3}}$ at the point where the curve meets the Y – axis.

20. Find the value of 'k', so that the length of the sub normal at any point on the curve $y = a^{1-k}x^k$ is a constant.

III. Long answer type questions **$5 \times 7 = 35$**

21. Find the circum centre of the triangle formed by the vertices (–2, 3), (2, –1) and (4, 0).

22. If Q (h, k) is the image of the point P (x_1, y_1) with respect to the line $ax + by + c = 0$, then prove that $\dfrac{h - x_1}{a} = \dfrac{k - y_1}{b} = \dfrac{-2(a\,x_1 + b\,y_1 + c)}{a^2 + b^2}$

Hence, find the image of the point (1, –2) w.r.t the straight line $2x - 3y + 5 = 0$.

23. If the equation $ax^2 + 2h\,xy + by^2 = 0$ represent a pair of intersecting lines, then the combined equation of the pair of bisectors of the angles between the lines is $h\,(x^2 - y^2) = (a - b)\,xy$.

24. Find the condition for the lines joining the origin to the points of intersection of the curve $x^2 + y^2 = a^2$ and the line $lx + my = 1$ to coincide.

25. The vertices of a triangle are A (1, 4, 2), B (–2, 1, 2), C (2, 3, –4). Find ∠A, ∠B and ∠C.

26. If $y = x\sqrt{a^2 + x^2} + a^2\log\left(x + \sqrt{a^2 + x^2}\right)$, then show that $\dfrac{dy}{dx} = 2\sqrt{a^2 + x^2}$.

27. If $\sin y = x\sin(a + y)$, then show that $\dfrac{dy}{dx} = \dfrac{\sin^2(a+y)}{\sin a}$

28. If the tangent at any point on the curve $x^m y^n = a^{m+n}$, meet the coordinate axes in A, B, then show that AP : PB is a constant.

29. Show that the condition for the orthogonality of the curves $ax^2 + b\,y^2 = 1$ and $a_1 x^2 + b_1 y^2 = 1$is $\dfrac{1}{a} - \dfrac{1}{b} = \dfrac{1}{a_1} - \dfrac{1}{b_1}$.

30. Find the maximum area of the rectangle that can be formed with fixed perimeter of the rectangle is 20.