SOLUTIONS OF SIMULTANEOUS EQUATIONS

Matrix Inversion Method:

1. Solve the following equations by matrix inversion method

$2x - y + z = 4,\ x + y + z = 1,\ x - 3y - 2z = 2$

Sol: Given equation are $2x - y + z = 4,\ x + y + z = 1,\ x - 3y - 2z = 2$

Matrix form of above equations is AX = B

Where $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & -3 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$AX = B \Longrightarrow X = A^{-1}B$

$$\det A = 2 \begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} - (-1) \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix}$$

$$= 2(-2 + 3) + 1(-2 - 1) + 1(-3 - 1)$$

$$= 2(1) + 1(-3) + 1(-4)$$

$$= 2 - 3 - 4 = -5 \neq 0$$

The cofactors of the elements of A are

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2 + 3 = 1$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ -3 & -2 \end{vmatrix} = -(-2 - 1) = 3$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} = -3 - 1 = -4$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ -3 & -2 \end{vmatrix} = -(2 + 3) = -5$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -4 - 1 = -5$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} = -(-6 + 1) = 5$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1 - 1 = -2$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = -(2 - 1) = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 + 1 = 3$$

$$\text{Cofactor matrix of } A = \begin{bmatrix} 1 & 3 & -4 \\ -5 & -5 & 5 \\ -2 & -1 & 3 \end{bmatrix}$$

$\text{Adj } A = (\text{cofactor matrix of } A)^T$

$$= \begin{bmatrix} 1 & -5 & -2 \\ 3 & -5 & -1 \\ -4 & 5 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{\det A} = -\frac{1}{5}\begin{bmatrix} 1 & -5 & -2 \\ 3 & -5 & -1 \\ -4 & 5 & 3 \end{bmatrix}$$

$$X = A^{-1}B = -\frac{1}{5}\begin{bmatrix} 1 & -5 & -2 \\ 3 & -5 & -1 \\ -4 & 5 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 1 \times 4 + (-5) \times 1 + (-2) \times 2 \\ 3 \times 4 + (-5) \times 1 + (-1) \times 2 \\ (-4) \times 4 + 5 \times 1 + 3 \times 2 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} 4-5-4 \\ 12-5-2 \\ -16+5+6 \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -5 \\ 5 \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$\therefore x = 1, y =- 1, z = 1$

2. Solve the following equations by matrix inversion method

$x + y + z = 6, x - y + z = 2, 2x + y - z = 1$

Sol: Given equation are $x + y + z = 6, x - y + z = 2, 2x + y - z = 1$

Matrix form of above equations is $AX = B$

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$

$AX = B \Longrightarrow X = A^{-1}B$

$\det A = 1\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$

$= 1(1- 1) - 1 (- 1 - 2) + 1 (1 + 2)$

$= 1(0) - 1 (- 3) + 1 (3)$

$= 0 +3+3 = 6 \neq 0$

The cofactors of the elements of A are

$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 1- 1 = 0$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = - (-1 - 2) = 3$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1+ 2 = 3$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = - (- 1 - 1) = 2$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = - 1 - 2 = - 3$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = - (1 - 2) = 1$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1 + 1 = 2$

$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = - (1 - 1) = 0$

$A_{33} = (-1)^{3+3}\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix} = -1-1 = -2$

Cofactor matrix of $A = \begin{bmatrix}0 & 3 & 3\\ 2 & -3 & 1\\ 2 & 0 & -2\end{bmatrix}$

Adj A = (cofactor matrix of A)^T

$= \begin{bmatrix}0 & 2 & 2\\ 3 & -3 & 0\\ 3 & 1 & -2\end{bmatrix}$

$A^{-1} = \frac{\text{adj }A}{\det A} = \frac{1}{6}\begin{bmatrix}0 & 2 & 2\\ 3 & -3 & 0\\ 3 & 1 & -2\end{bmatrix}$

$X = A^{-1}B = \frac{1}{6}\begin{bmatrix}0 & 2 & 2\\ 3 & -3 & 0\\ 3 & 1 & -2\end{bmatrix}\begin{bmatrix}6\\ 2\\ 1\end{bmatrix} = \frac{1}{6}\begin{bmatrix}0\times 6+2\times 2+2\times 1\\ 3\times 6+(-3)\times 2+0\times 1\\ 3\times 6+1\times 2+(-2)\times 1\end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix}0+4+2\\ 18-6+0\\ 18+2-2\end{bmatrix} = \frac{1}{6}\begin{bmatrix}6\\ 12\\ 18\end{bmatrix} = \begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$

$\therefore x = 1, y = 2, z = 3$

3. Solve the following equations by matrix inversion method

$3x + y + 2z = 3,\ 2x - 3y - z = -3,\ x + 2y + z = 4$

Sol: Given equation are $3x + y + 2z = 3,\ 2x - 3y - z = -3,\ x + 2y + z = 4$

Matrix form of above equations is AX = B

Where $A = \begin{bmatrix}3 & 1 & 2\\ 2 & -3 & -1\\ 1 & 2 & 1\end{bmatrix}, X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}, B = \begin{bmatrix}3\\ -3\\ 4\end{bmatrix}$

$AX = B \Longrightarrow X = A^{-1}B$

$\det A = 3\begin{vmatrix}-3 & -1\\ 2 & 1\end{vmatrix} - 1\begin{vmatrix}2 & -1\\ 1 & 1\end{vmatrix} + 2\begin{vmatrix}2 & -3\\ 1 & 2\end{vmatrix}$

$= 3(-3+2) - 1(2+1) + 2(4+3)$

$= 3(-1) - 1(3) + 2(7)$

$= -3 - 3 + 14 = 8 \neq 0$

The cofactors of the elements of A are

$A_{11} = (-1)^{1+1}\begin{vmatrix}-3 & -1\\ 2 & 1\end{vmatrix} = -3+2 = -1$

$A_{12} = (-1)^{1+2}\begin{vmatrix}2 & -1\\ 1 & 1\end{vmatrix} = -(2+1) = -3$

$A_{13} = (-1)^{1+3}\begin{vmatrix}2 & -3\\ 1 & 2\end{vmatrix} = 4+3 = 7$

$A_{21} = (-1)^{2+1}\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix} = -(1-4) = 3$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = -(6-1) = -5$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 2 \\ -3 & -1 \end{vmatrix} = -1 + 6 = 5$

$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -(-3-4) = 7$

$A_{33} = (-1)^{3+3}\begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} = -9 - 2 = -11$

Cofactor matrix of A $= \begin{bmatrix} -1 & -3 & 7 \\ 3 & 1 & -5 \\ 5 & 7 & -11 \end{bmatrix}$

Adj A = (cofactor matrix of A)^T

$= \begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$

$A^{-1} = \frac{\text{adj } A}{\det A} = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$

$X = A^{-1}B = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} (-1)\times 3 + 3\times(-3) + 5\times 4 \\ (-3)\times 3 + 1\times(-3) + 7\times 4 \\ 7\times 3 + (-5)\times(-3) - 11\times 4 \end{bmatrix}$

$= \frac{1}{8}\begin{bmatrix} -3 - 9 + 20 \\ -9 - 3 + 28 \\ 21 + 15 - 44 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$\therefore x = 1, y = 2, z = -1$

4. Solve the following equations by matrix inversion method

$2x + y - z = 1, x + y - z = 0, 3x + 2y + 2z = 5$

Sol: Given equation are $2x + y - z = 1, x + y - z = 0, 3x + 2y + 2z = 5$

Matrix form of above equations is AX = B

Where $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -1 \\ 3 & 2 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

$AX = B \Longrightarrow X = A^{-1}B$

$\det A = 2\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$

$= 2(2+2) - 1(2+3) - 1(2-3)$

$= 2(4) - 1(5) - 1(-1)$

$= 8 - 5 + 1 = 4 \neq 0$

The cofactors of the elements of A are

$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = 2 + 2 = 4$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = -(2 + 3) = -5$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} = 2 - 3 = -1$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} = -(2 + 2) = -4$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4 + 3 = 7$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = -(4 - 3) = -1$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} = -1 + 1 = 0$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -(-2 + 1) = 1$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1$

Cofactor matrix of $A = \begin{bmatrix} 4 & -5 & -1 \\ -4 & 7 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

Adj $A = (\text{cofactor matrix of } A)^T$

$$= \begin{bmatrix} 4 & -4 & 0 \\ -5 & 7 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{\det A} = \frac{1}{4}\begin{bmatrix} 4 & -4 & 0 \\ -5 & 7 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 4 & -4 & 0 \\ -5 & 7 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \times 1 + (-4) \times 0 + \ 0 \times 5 \\ (-5) \times 1 + 7 \times 0 + 1 \times 5 \\ (-1) \times 1 + (-1) \times 0 + 1 \times 5 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 + 0 + 0 \\ -5 + 0 + 5 \\ -1 + 0 + 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore x = 1, y = 0, z = 1$

5. Solve the following equations by matrix inversion method

$x + 2y + 3z = 6,\ 2x + 4y + z = 7,\ 3x + 2y + 3z = 8$

Sol: Given equation are $x + 2y + 3z = 6,\ 2x + 4y + z = 7,\ 3x + 2y + 3z = 8$

Matrix form of above equations is $AX = B$

Where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$$\det A = 1\begin{vmatrix}4 & 1\\2 & 3\end{vmatrix} - 2\begin{vmatrix}2 & 1\\3 & 3\end{vmatrix} + 3\begin{vmatrix}2 & 4\\3 & 2\end{vmatrix}$$

$= 1(12 - 2) - 2\,(6 - 3) + 3\,(4 - 12)$

$= 1(10) - 2\,(3) + 3\,(-8)$

$= 10 - 6 - 24 = -20 \neq 0$

The cofactors of the elements of A are

$A_{11} = (-1)^{1+1}\begin{vmatrix}4 & 1\\2 & 3\end{vmatrix} = 12 - 2 = 10$

$A_{12} = (-1)^{1+2}\begin{vmatrix}2 & 1\\3 & 3\end{vmatrix} = -(6 - 3) = -3$

$A_{13} = (-1)^{1+3}\begin{vmatrix}2 & 4\\3 & 2\end{vmatrix} = 4 - 12 = -8$

$A_{21} = (-1)^{2+1}\begin{vmatrix}2 & 3\\2 & 3\end{vmatrix} = -(6 - 6) = 0$

$A_{22} = (-1)^{2+2}\begin{vmatrix}1 & 3\\3 & 3\end{vmatrix} = 3 - 9 = -6$

$A_{23} = (-1)^{2+3}\begin{vmatrix}1 & 2\\3 & 2\end{vmatrix} = -(2 - 6) = 4$

$A_{31} = (-1)^{3+1}\begin{vmatrix}2 & 3\\4 & 1\end{vmatrix} = 2 - 12 = -10$

$A_{32} = (-1)^{3+2}\begin{vmatrix}1 & 3\\2 & 1\end{vmatrix} = -(1 - 6) = 5$

$A_{33} = (-1)^{3+3}\begin{vmatrix}1 & 2\\2 & 4\end{vmatrix} = 4 - 4 = 0$

$$\text{Cofactor matrix of A} = \begin{bmatrix}10 & -3 & -8\\0 & -6 & 4\\-10 & 5 & 0\end{bmatrix}$$

Adj A = (cofactor matrix of A)^T

$$= \begin{bmatrix}10 & 0 & -10\\-3 & -6 & 5\\-8 & 4 & 0\end{bmatrix}$$

$$A^{-1} = \frac{\text{adj A}}{\det A} = -\frac{1}{20}\begin{bmatrix}10 & 0 & -10\\-3 & -6 & 5\\-8 & 4 & 0\end{bmatrix}$$

$$X = A^{-1}B = -\frac{1}{20}\begin{bmatrix}10 & 0 & -10\\-3 & -6 & 5\\-8 & 4 & 0\end{bmatrix}\begin{bmatrix}6\\7\\8\end{bmatrix} = -\frac{1}{20}\begin{bmatrix}10\times 6 + 0\times 7 + (-10)\times 8\\(-3)\times 6 + (-6)\times 7 + 5\times 8\\(-8)\times 6 + 4\times 7 + 0\times 8\end{bmatrix}$$

$$= -\frac{1}{20}\begin{bmatrix}60 - 0 - 80\\-18 - 42 + 40\\-48 + 28 + 0\end{bmatrix} = -\frac{1}{20}\begin{bmatrix}-20\\-20\\-20\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\therefore x = 1, y = 1, z = 1$

6. Solve the following equations by matrix inversion method

x + y + z = 1, 2x + 2y + 3z = 6, x + 4y + 9z = 3

Sol: Given equation are x + y + z = 1, 2x + 2y + 3z = 6, x + 4y + 9z = 3

Matrix form of above equations is AX = B

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$

$AX = B \Longrightarrow X = A^{-1}B$

$\det A = 1\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ 1 & 9 \end{vmatrix} + 1\begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix}$

$= 1(18 - 12) - 1\,(18 - 3) + 1\,(8 - 2)$

$= 1(6) - 2\,(15) + 3\,(6)$

$= 6 - 30 + 18 = -6 \neq 0$

The cofactors of the elements of A are

$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 18 - 12 = 6$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 3 \\ 1 & 9 \end{vmatrix} = -(18 - 3) = -15$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} = 8 - 2 = 6$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -(9 - 4) = -5$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 9 - 1 = 8$ $\qquad \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -(4 - 1) = -3$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3 - 2 = 1$

$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -(3 - 2) = -1$

$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2 - 2 = 0$

Cofactor matrix of $A = \begin{bmatrix} 6 & -15 & 6 \\ -5 & 8 & -3 \\ 1 & -1 & 0 \end{bmatrix}$

Adj A = (cofactor matrix of A)^T

$= \begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj } A}{\det A} = -\frac{1}{6}\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}$$

$$X = A^{-1}B = -\frac{1}{6}\begin{bmatrix} 6 & -5 & 1 \\ -15 & 8 & -1 \\ 6 & -3 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} = -\frac{1}{6}\begin{bmatrix} 6 \times 1 + (-5) \times 6 + 1 \times 3 \\ (-15) \times 1 + 8 \times 6 + (-1) \times 3 \\ 6 \times 1 + (-3) \times 6 + 0 \times 3 \end{bmatrix}$$

$$= -\frac{1}{6}\begin{bmatrix} 60 - 0 - 80 \\ -18 - 42 + 40 \\ -48 + 28 + 0 \end{bmatrix} = -\frac{1}{6}\begin{bmatrix} -20 \\ -20 \\ -20 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore x = 7, y = -10, z = 4$

Cramer's Rule:

7. Solve the following equations by Cramer's Rule

$x + y + z = 9,\ 2x + 5y + 7z = 52,\ 2x + 5y - z = 0$

Sol: Given equation are $x + y + z = 9,\ 2x + 5y + 7z = 52,\ 2x + 5y - z = 0$

Matrix form of above equations is AX = B

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 5 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} = 1\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$$= 1(-5 - 7) - 1(-2 - 14) + (2 - 10) = -12 + 16 - 8 = -4$$

$$\Delta_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 0 & 1 & -1 \end{vmatrix} = 9\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 52 & 5 \\ 0 & 1 \end{vmatrix}$$

$$= 9(-5 - 7) - 1(-52 - 0) + 1(52 - 0)$$

$$= 9(-12) + 1(52) + 52$$

$$= -108 + 52 + 52 = -4$$

$$\Delta_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix} = 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} - 9\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix}$$

$$= 1(-52 - 0) - 9(-2 - 14) + 1(0 - 104)$$

$$= 1(-5\ 2) - 9(-16) + (-104)$$

$$= -52 + 144 - 104 = -156 + 144 = -12$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 2 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 5 & 52 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix} + 9\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(0 - 52) - 1\ (0 - 104) + 9(2 - 10) = -12 + 16 - 8 = -4$

$= 1(-5\ 2) - 1\ (-104) + 9(-8)$

$= -52 + 104 - 72 = -124 + 104 = -20$

$x = \frac{\Delta_1}{\Delta} = \frac{-4}{-4} = 1 \qquad y = \frac{\Delta_2}{\Delta} = \frac{-12}{-4} = 3 \qquad z = \frac{\Delta_3}{\Delta} = \frac{-20}{-4} = 5$

$\therefore x = 1, y = 3, z = 5$

8. Solve the following equations by Cramer's Rule

$x + 2y - z = -3,\ 3x + y + z = 4,\ x - y + 2z = 6$

Sol: Given equation are $x + 2y - z = -3,\ 3x + y + z = 4,\ x - y + 2z = 6$

Matrix form of above equations is $AX = B$

Where $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -3 \\ 4 \\ 6 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} = 1\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(-5 - 7) - 1\ (-2 - 14) + (2 - 10) = -12 + 16 - 8 = -4$

$$\Delta_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 0 & 1 & -1 \end{vmatrix} = 9\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 52 & 5 \\ 0 & 1 \end{vmatrix}$$

$= 9(-5 - 7) - 1\ (-52 - 0) + 1(52 - 0)$

$= 9(-12) + 1(52) + 52$

$= -108 + 52 + 52 = -4$

$$\Delta_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix} = 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} - 9\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix}$$

$= 1(-52 - 0) - 9\ (-2 - 14) + 1(0 - 104)$

$= 1(-5\ 2) - 9\ (-16) + (-104)$

$= -52 + 144 - 104 = -156 + 144 = -12$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 2 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 5 & 52 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix} + 9\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(0 - 52) - 1\ (0 - 104) + 9(2 - 10) = -12 + 16 - 8 = -4$

$= 1(-5\ 2) - 1\ (-104) + 9(-8)$

$= -52 + 104 - 72 = -124 + 104 = -20$

$x = \frac{\Delta_1}{\Delta} = \frac{-4}{-4} = 1 \qquad y = \frac{\Delta_2}{\Delta} = \frac{-12}{-4} = 3 \qquad z = \frac{\Delta_3}{\Delta} = \frac{-20}{-4} = 5$

$\therefore x = 1, y = 3, z = 5$

9. Solve the following equations by Cramer's Rule

$x + 2y - z = -1, 3x - y - 2z = 5, x - y - 3z = 0$

Sol: Given equation are $x + 2y - z = -1, 3x - y - 2z = 5, x - y - 3z = 0$

Matrix form of above equations is AX = B

$$\text{Where } A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & -2 \\ 1 & -1 & -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & -2 \\ 1 & -1 & -3 \end{vmatrix} = 1\begin{vmatrix} -1 & -2 \\ -1 & -3 \end{vmatrix} - 2\begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -1 \\ 1 & -1 \end{vmatrix}$$

$$= 1(3 - 2) - 2(-9 + 2) + (-1)(-3 + 1) = 1 + 14 + 2 = 17$$

$$\Delta_1 = \begin{vmatrix} -1 & 2 & -1 \\ 5 & -1 & -2 \\ 0 & -1 & -3 \end{vmatrix} = (-1)\begin{vmatrix} -1 & -2 \\ -1 & -3 \end{vmatrix} - 2\begin{vmatrix} 5 & -2 \\ 0 & -3 \end{vmatrix} + (-1)\begin{vmatrix} 5 & -1 \\ 0 & -1 \end{vmatrix}$$

$$= (-1)(3 - 2) - 2(-15 - 0) + (-1)(-5 - 0)$$

$$= (-1)(1) - (-15) + 5$$

$$= -1 + 15 + 5 = -4$$

$$\Delta_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix} = 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} - 9\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix}$$

$$= 1(-52 - 0) - 9(-2 - 14) + 1(0 - 104)$$

$$= 1(-52) - 9(-16) + (-104)$$

$$= -52 + 144 - 104 = -156 + 144 = -12$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & -2 \\ 1 & -1 & -3 \end{vmatrix} = 1\begin{vmatrix} 5 & 52 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix} + 9\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$$= 1(0 - 52) - 1(0 - 104) + 9(2 - 10) = -12 + 16 - 8 = -4$$

$$= 1(-52) - 1(-104) + 9(-8)$$

$$= -52 + 104 - 72 = -124 + 104 = -20$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-4}{-4} = 1 \qquad y = \frac{\Delta_2}{\Delta} = \frac{-12}{-4} = 3 \qquad z = \frac{\Delta_3}{\Delta} = \frac{-20}{-4} = 5$$

$\therefore x = 2, y = -1, z = 1$

10. Solve the following equations by Cramer's Rule

$x + y + z = 8, 3x + 5y - 7z = 14, x - y - 3z = 0$

Sol: Given equation are $x + y + z = 9, 2x + 5y + 7z = 52, 2x + 5y - z = 0$

Matrix form of above equations is AX = B

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 5 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} = 1\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(-5-7) - 1(-2-14) + (2-10) = -12 + 16 - 8 = -4$

$$\Delta_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 0 & 1 & -1 \end{vmatrix} = 9\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 52 & 5 \\ 0 & 1 \end{vmatrix}$$

$= 9(-5-7) - 1(-52-0) + 1(52-0)$

$= 9(-12) + 1(52) + 52$

$= -108 + 52 + 52 = -4$

$$\Delta_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix} = 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} - 9\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix}$$

$= 1(-52-0) - 9(-2-14) + 1(0-104)$

$= 1(-52) - 9(-16) + (-104)$

$= -52 + 144 - 104 = -156 + 144 = -12$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 2 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 5 & 52 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix} + 9\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(0-52) - 1(0-104) + 9(2-10) = -12 + 16 - 8 = -4$

$= 1(-52) - 1(-104) + 9(-8)$

$= -52 + 104 - 72 = -124 + 104 = -20$

$x = \frac{\Delta_1}{\Delta} = \frac{-4}{-4} = 1 \qquad y = \frac{\Delta_2}{\Delta} = \frac{-12}{-4} = 3 \qquad z = \frac{\Delta_3}{\Delta} = \frac{-20}{-4} = 5$

$\therefore x = 5, y = 5/3, z = 4/3$

11. Solve the following equations by Cramer's Rule

$x + 2y + 3z = 14,\ 3x + y + 2z = 11,\ 2x + 3y + z = 11$

Sol: Given equation are $x + y + z = 9,\ 2x + 5y + 7z = 52,\ 2x + 5y - z = 0$

Matrix form of above equations is AX = B

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 5 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} = 1\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(-5-7) - 1(-2-14) + (2-10) = -12 + 16 - 8 = -4$

$$\Delta_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 0 & 1 & -1 \end{vmatrix} = 9\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 52 & 5 \\ 0 & 1 \end{vmatrix}$$

$= 9(-5-7) - 1(-52-0) + 1(52-0)$

$= 9(-12) + 1(52) + 52$

$= -108 + 52 + 52 = -4$

$$\Delta_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix} = 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} - 9\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix}$$

$= 1(-52-0) - 9(-2-14) + 1(0-104)$

$= 1(-52) - 9(-16) + (-104)$

$= -52 + 144 - 104 = -156 + 144 = -12$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 2 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 5 & 52 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix} + 9\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(0-52) - 1(0-104) + 9(2-10) = -12 + 16 - 8 = -4$

$= 1(-52) - 1(-104) + 9(-8)$

$= -52 + 104 - 72 = -124 + 104 = -20$

$x = \frac{\Delta_1}{\Delta} = \frac{-4}{-4} = 1 \qquad y = \frac{\Delta_2}{\Delta} = \frac{-12}{-4} = 3 \qquad z = \frac{\Delta_3}{\Delta} = \frac{-20}{-4} = 5$

$\therefore x = 1, y = 2, z = 3$

12. Solve the following equations by Cramer's Rule

$x + 2y + 3z = 6, 3x - 2y + 4z = 5, x - y - z = -1$

Sol: Given equation are $x + y + z = 9, 2x + 5y + 7z = 52, 2x + 5y - z = 0$

Matrix form of above equations is AX = B

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 5 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix} = 1\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(-5-7) - 1(-2-14) + (2-10) = -12 + 16 - 8 = -4$

$$\Delta_1 = \begin{vmatrix} 9 & 1 & 1 \\ 52 & 5 & 7 \\ 0 & 1 & -1 \end{vmatrix} = 9\begin{vmatrix} 5 & 7 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 52 & 5 \\ 0 & 1 \end{vmatrix}$$

$= 9(-5-7) - 1(-52-0) + 1(52-0)$

$= 9(-12) + 1(52) + 52$

$= -108 + 52 + 52 = -4$

$$\Delta_2 = \begin{vmatrix} 1 & 9 & 1 \\ 2 & 52 & 7 \\ 2 & 0 & -1 \end{vmatrix} = 1\begin{vmatrix} 52 & 7 \\ 0 & -1 \end{vmatrix} - 9\begin{vmatrix} 2 & 7 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix}$$

$= 1(-52 - 0) - 9(-2 - 14) + 1(0 - 104)$

$= 1(-5\ 2) - 9(-16) + (-104)$

$= -52 + 144 - 104 = -156 + 144 = -12$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 9 \\ 2 & 5 & 52 \\ 2 & 1 & 0 \end{vmatrix} = 1\begin{vmatrix} 5 & 52 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 52 \\ 2 & 0 \end{vmatrix} + 9\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$$

$= 1(0 - 52) - 1(0 - 104) + 9(2 - 10) = -12 + 16 - 8 = -4$

$= 1(-5\ 2) - 1(-104) + 9(-8)$

$= -52 + 104 - 72 = -124 + 104 = -20$

$x = \frac{\Delta_1}{\Delta} = \frac{-4}{-4} = 1 \qquad y = \frac{\Delta_2}{\Delta} = \frac{-12}{-4} = 3 \qquad z = \frac{\Delta_3}{\Delta} = \frac{-20}{-4} = 5$

$\therefore x = 1, y = 1, z = 1$

Gauss – Jordan Method:

13. Solve the following equations by using Gauss – Jordan method

$3x + y + 2z = 6, x + 2y + z = 4, x + y - 2z = 2$

Sol: Given equation are $3x + y + 2z = 6, x + 2y + z = 4, x + y - 2z = 2$

Matrix form of above equations is $AX = B$

Where $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

Augmented matrix of $A = \begin{bmatrix} 3 & 1 & 2 & 6 \\ 1 & 2 & 1 & 4 \\ 1 & 1 & -2 & 2 \end{bmatrix}$

$\therefore x = 1, y = 1, z = 1$

14. Solve the following equations by Cramer's Rule

$x + y + z = 6, x + 2y + 3z = 14, x + 4y + 9z = 36$

Sol: Given equation are $x + y + z = 6, x + 2y + 3z = 14, x + 4y + 9z = 36$

Matrix form of above equations is $AX = B$

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 14 \\ 36 \end{bmatrix}$

Augmented matrix of $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & 5 & 7 & 14 \\ 1 & 4 & 9 & 36 \end{bmatrix}$

$\therefore x = 1, y = 2, z = 3$

15. Solve the following equations by using Gauss – Jordan method

$x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$

Sol: Given equation are $x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6$

Matrix form of above equations is $AX = B$

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 5 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$

Augmented matrix of $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 5 & -1 & 6 \end{bmatrix}$

$\therefore x = 2, y = 1, z = 0$

16. Solve the following equations by using Gauss – Jordan method

$x + y - z = 0, 2x + y - z = 1, 3x + 2y + 2z = 5$

Sol: Given equation are $x + y - z = 0, 2x + y - z = 1, 3x + 2y + 2z = 5$

Matrix form of above equations is $AX = B$

Where $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & -1 \\ 3 & 2 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$

Augmented matrix of $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 2 & 1 & -1 & 1 \\ 3 & 2 & 2 & 5 \end{bmatrix}$

$\therefore x = 1, y = 0, z = 1$

17. Solve the following equations by using Gauss – Jordan method

$2x + 3y + z = 11, x + 2y + z = 8, 3x - y - 2z = -5$

Sol: Given equation are $2x + 3y + z = 11$, $x + 2y + z = 8$, $3x - y - 2z = -5$

Matrix form of above equations is $AX = B$

Where $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 11 \\ 8 \\ -5 \end{bmatrix}$

Augmented matrix of $A = \begin{bmatrix} 2 & 3 & 1 & 11 \\ 1 & 2 & 1 & 8 \\ 3 & -1 & -2 & -5 \end{bmatrix}$

$\therefore x = 1, y = 2, z = 3$