TRANSFORMATIONS – PART 2

1. If $A + B + C = 180^0$, then prove that $\text{Sin } 2A + \text{Sin } 2B + \text{Sin } 2C = 4 \text{ Sin } A \text{ Sin } B \text{ Sin } C$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$\text{Sin } (A + B) = \text{Sin } (180^0 - C) = \text{Sin } C$

$\text{Cos } (A + B) = \text{Cos } (180^0 - C) = - \text{Cos } C$

$\text{Sin } (A + B) = \text{Sin } C$ ---------------- (1)

$\text{Cos } (A + B) = - \text{Cos } C$---------------- (2)

$\text{LHS} = \text{Sin } 2A + \text{Sin } 2B + \text{Sin } 2C$

$= 2\text{Sin}\left(\frac{2A + 2B}{2}\right) \text{Cos}\left(\frac{2A - 2B}{2}\right) + \text{Sin } 2C$

$\boxed{\text{Sin } C + \text{Sin } D = 2 \text{ Sin}\left(\frac{C + D}{2}\right) \text{Cos}\left(\frac{C - D}{2}\right)}$

$= 2\text{Sin } (A + B) \text{ Cos } (A - B) + \text{Sin } 2C$

$= 2\text{Sin } C \text{ Cos } (A - B) + 2 \text{ Sin } C \text{ Cos } C$ [From (1)]

$= 2 \text{ Sin } C [\text{ Cos } (A - B) + \text{Cos } C]$

$= 2 \text{ Sin } C [\text{ Cos } (A - B) - \text{Cos } (A + B)]$ [From (2)]

$= 2 \text{ Sin } C [2 \text{ Sin } A \text{ Sin } B]$

$\boxed{\text{Cos } (A - B) - \text{Cos } (A + B) = 2 \text{ Sin } A \text{ Sin } B}$

$= 4 \text{ Sin } A \text{ Sin } B \text{ Sin } C = \text{RHS}$

2. If $A + B + C = 180^0$, then prove that $\text{Sin } 2A + \text{Sin } 2B - \text{Sin } 2C = 4 \text{ Cos } A \text{ Cos } B \text{ Sin } C$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$\text{Sin } (A + B) = \text{Sin } (180^0 - C) = \text{Sin } C$

$\text{Cos } (A + B) = \text{Cos } (180^0 - C) = - \text{Cos } C$

$\text{Sin } (A + B) = \text{Sin } C$ ---------------- (1)

$\text{Cos } (A + B) = - \text{Cos } C$---------------- (2)

$\text{LHS} = \text{Sin } 2A + \text{Sin } 2B - \text{Sin } 2C$

$= 2\text{Sin}\left(\frac{2A + 2B}{2}\right) \text{Cos}\left(\frac{2A - 2B}{2}\right) - \text{Sin } 2C$

$\boxed{\text{Sin } C + \text{Sin } D = 2 \text{ Sin}\left(\frac{C + D}{2}\right) \text{Cos}\left(\frac{C - D}{2}\right)}$

$= 2\text{Sin } (A + B) \text{ Cos } (A - B) - \text{Sin } 2C$

$= 2\text{Sin } C \text{ Cos } (A - B) - 2 \text{ Sin } C \text{ Cos } C$ [From (1)]

$= 2 \text{ Sin C} [\text{ Cos }(A - B) - \text{Cos C}]$

$= 2 \text{ Sin C} [\text{ Cos }(A - B) + \text{Cos }(A + B)]$ [From (2)]

$= 2 \text{ Sin C} [\text{ 2 Cos A Cos B}]$

$\text{Cos }(A + B) + \text{Cos }(A - B) = 2 \text{ Cos A Cos B}$

$= 4 \text{ Cos A Cos B Sin C} = \text{RHS}$

3. If $A + B + C = 180^0$, then prove that $\text{Sin } 2A - \text{Sin } 2B + \text{Sin } 2C = 4 \text{ Cos A Sin B Cos C}$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$\text{Sin }(A + B) = \text{Sin }(180^0 - C) = \text{Sin C}$

$\text{Cos }(A + B) = \text{Cos }(180^0 - C) = -\text{Cos C}$

$\text{Sin }(A + B) = \text{Sin C}$ ---------------- (1)

$\text{Cos }(A + B) = -\text{Cos C}$---------------- (2)

$\text{LHS} = \text{Sin } 2A - \text{Sin } 2B + \text{Sin } 2C$

$= 2 \text{ Cos}\left(\frac{2A + 2B}{2}\right) \text{Sin}\left(\frac{2A - 2B}{2}\right) + \text{Sin } 2C$

$\text{Sin C} - \text{Sin D} = 2 \text{ Cos}\left(\frac{C + D}{2}\right) \text{Sin}\left(\frac{C - D}{2}\right)$

$= 2\text{Cos }(A + B) \text{ Sin }(A - B) + \text{Sin } 2C$

$= -2\text{Cos C Sin }(A - B) + 2 \text{ Sin C Cos C}$ [From (2)]

$= 2 \text{ Cos C} [-\text{Sin }(A - B) + \text{Sin C}]$

$= 2 \text{ Cos C} [\text{ Sin }(A + B) - \text{Sin }(A - B)]$ [From (1)]

$= 2 \text{ Cos C} [\text{ 2 Cos A Sin B}]$

$\text{Sin }(A + B) - \text{sin }(A - B) = 2 \text{ Cos A Sin B}$

$= 4 \text{ Cos A Sin B Cos C} = \text{RHS}$

4. If $A + B + C = 180^0$, then prove that $\text{Sin } 2A - \text{Sin } 2B - \text{Sin } 2C = -4 \text{ Sin A Cos B Cos C}$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$\text{Sin }(A + B) = \text{Sin }(180^0 - C) = \text{Sin C}$

$\text{Cos }(A + B) = \text{Cos }(180^0 - C) = -\text{Cos C}$

$\text{Sin }(A + B) = \text{Sin C}$ ---------------- (1)

$\text{Cos }(A + B) = -\text{Cos C}$---------------- (2)

$\text{LHS} = \text{Sin } 2A - \text{Sin } 2B - \text{Sin } 2C$

$= 2 \text{ Cos}\left(\frac{2A + 2B}{2}\right) \text{Sin}\left(\frac{2A - 2B}{2}\right) - \text{Sin } 2C$

$\text{Sin C} - \text{Sin D} = 2 \text{ Cos}\left(\frac{C + D}{2}\right) \text{Sin}\left(\frac{C + D}{2}\right)$

$= 2\text{Cos}\ (A + B)\ \text{Sin}\ (A - B) - \text{Sin}\ 2C$

$= -\ 2\text{Cos}\ C\ \text{Sin}\ (A - B) - 2\ \text{Sin}\ C\ \text{Cos}\ C$ [From (2)]

$= 2\ \text{Cos}\ C\ [-\ \text{Sin}\ (A - B) - \text{Sin}\ C]$

$= 2\ \text{Cos}\ C\ [-\ \text{Sin}\ (A - B) - \text{Sin}\ (A + B)]$ [From (1)]

$= -2\ \text{Cos}\ C\ [\text{Sin}\ (A + B) + \text{Sin}\ (A - B)]$

Sin (A + B) + sin (A − B) = 2 Sin A Cos B

$= -2\ \text{Cos}\ C\ [\ 2\ \text{Sin}\ A\ \text{Cos}\ B]$

$= -4\ \text{Sin}\ A\ \text{Cos}\ B\ \text{Cos}\ C = \text{RHS}$

5. If A, B, C are the angles of a Triangle, then prove that

$\text{Cos}\ 2A + \text{Cos}\ 2B + \text{Cos}\ 2C = -1 - 4\ \text{Cos}\ A\ \text{Cos}\ B\ \text{Cos}\ C$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$\text{Sin}\ (A + B) = \text{Sin}\ (180^0 - C) = \text{Sin}\ C$

$\text{Cos}\ (A + B) = \text{Cos}\ (180^0 - C) = -\ \text{Cos}\ C$

$\text{Sin}\ (A + B) = \text{Sin}\ C$ ---------------- (1)

$\text{Cos}\ (A + B) = -\ \text{Cos}\ C$---------------- (2)

$\text{LHS} = \text{Cos}\ 2A + \text{Cos}\ 2B + \text{Cos}\ 2C$

$= 2\ \text{Cos}\left(\frac{2A + 2B}{2}\right) \text{Cos}\left(\frac{2A - 2B}{2}\right) + \text{Cos}\ 2C$

$\text{Cos}\ C + \text{Cos}\ D = 2\ \text{Cos}\left(\frac{C + D}{2}\right) \text{Cos}\left(\frac{C + D}{2}\right)$

$= 2\text{Cos}\ (A + B)\ \text{Cos}\ (A - B) + \text{Cos}\ 2C$

$= -\ 2\text{Cos}\ C\ \text{Cos}\ (A - B) + 2\ \text{Cos}^2\ C - 1$ [From (2)]

$= -\ 1 - 2\ \text{Cos}\ C\ [\ \text{Cos}\ (A - B) - \text{Cos}\ C]$

$= -1 + 2\ \text{Cos}\ C\ [\ \text{Cos}\ (A - B) + \text{Cos}\ (A + B)]$ [From (2)]

$= -1 - 2\ \text{Cos}\ C\ [\ 2\ \text{Cos}\ A\ \text{Cos}\ B]$

Cos (A + B) + Cos (A − B) = 2 Cos A Cos B

$= -1 - 4\ \text{Cos}\ A\ \text{Cos}\ B\ \text{Cos}\ C = \text{RHS}$

6. If $A + B + C = 180^0$, then prove that $\text{Cos}\ 2A + \text{Cos}\ 2B - \text{Cos}\ 2C = 1 - 4\ \text{Sin}\ A\ \text{Sin}\ B\ \text{Cos}\ C$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$\text{Sin}\ (A + B) = \text{Sin}\ (180^0 - C) = \text{Sin}\ C$

$\text{Cos}\ (A + B) = \text{Cos}\ (180^0 - C) = -\ \text{Cos}\ C$

$\text{Sin}(A+B) = \text{Sin } C$ ---------------- (1)

$\text{Cos}(A+B) = -\text{Cos } C$---------------- (2)

LHS = $\text{Cos } 2A + \text{Cos } 2B - \text{Cos } 2C$

$= 2\,\text{Cos}\left(\frac{2A+2B}{2}\right)\text{Cos}\left(\frac{2A-2B}{2}\right) - \text{Cos } 2C$

$\text{Cos } C + \text{Cos } D = 2\,\text{Cos}\left(\frac{C+D}{2}\right)\text{Cos}\left(\frac{C+D}{2}\right)$

$= 2\text{Cos}(A+B)\,\text{Cos}(A-B) - \text{Cos } 2C$

$= -2\text{Cos } C\,\text{Cos}(A-B) - (2\,\text{Cos}^2 C - 1)$ [From (2)]

$\text{Cos } 2\theta = 2\,\text{Cos}^2\theta - 1$

$= -2\text{Cos } C\,\text{Cos}(A-B) - 2\,\text{Cos}^2 C + 1$

$= 1 - 2\,\text{Cos } C\,[\text{Cos}(A-B) + \text{Cos } C]$

$= 1 + 2\,\text{Cos } C\,[\text{Cos}(A-B) - \text{Cos}(A+B)]$ [From (2)]

$= 1 - 2\,\text{Cos } C\,[2\,\text{Sin } A\,\text{Sin } B]$

$\text{Cos}(A-B) - \text{Cos}(A+B) = 2\,\text{Sin } A\,\text{Sin } B$

$= 1 - 4\,\text{Sin } A\,\text{Sin } B\,\text{Cos } C$ = RHS

7. If $A + B + C = 180^0$, then prove that $\text{Cos } 2A - \text{Cos } 2B + \text{Sin } 2C = 1 - 4\,\text{Sin } A\,\text{Cos } B\,\text{Sin } C$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$\text{Sin}(A+B) = \text{Sin}(180^0 - C) = \text{Sin } C$

$\text{Cos}(A+B) = \text{Cos}(180^0 - C) = -\text{Cos } C$

$\text{Sin}(A+B) = \text{Sin } C$ ---------------- (1)

$\text{Cos}(A+B) = -\text{Cos } C$---------------- (2)

LHS = $\text{Cos } 2A - \text{Cos } 2B + \text{Cos } 2C$

$= -2\,\text{Sin}\left(\frac{2A+2B}{2}\right)\text{Sin}\left(\frac{2A-2B}{2}\right) + \text{Cos } 2C$

$\text{Cos } C - \text{Cos } D = -2\text{Sin}\left(\frac{C+D}{2}\right)\text{Sin}\left(\frac{C+D}{2}\right)$

$= -2\text{Sin}(A+B)\,\text{Sin}(A-B) + \text{Cos } 2C$

$= -2\,\text{Sin } C\,\text{Sin}(A-B) + 1 - 2\,\text{Sin}^2 C$ [From (1)]

$\text{Cos } 2\theta = 1 - 2\,\text{Sin}^2\theta$

$= 1 - 2\,\text{Sin } C\,[\text{Sin}(A-B) + \text{Sin } C]$

$= 1 + 2\,\text{Sin } C\,[\text{Sin}(A-B) + \text{Sin}(A+B)]$ [From (1)]

$= 1 - 2\,\text{Sin } C\,[2\,\text{Sin } A\,\text{Cos } B]$

$\text{Sin}(A+B) + \text{Sin}(A-B) = 2\,\text{Sin } A\,\text{Cos } B$

$= 1 - 4\,\text{Sin } A\,\text{Cos } B\,\text{Sin } C$ = RHS

8. If $A + B + C = 180^0$, then prove that $\text{Sin}^2 A + \text{Sin}^2 B + \text{Sin}^2 C = 2(1 + \text{Cos } A \text{ Cos } B \text{ Cos } C)$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$\text{Sin } (A + B) = \text{Sin } (180^0 - C) = \text{Sin } C$

$\text{Cos } (A + B) = \text{Cos } (180^0 - C) = - \text{Cos } C$

$\text{Sin } (A + B) = \text{Sin } C$ ---------------- (1)

$\text{Cos } (A + B) = - \text{Cos } C$---------------- (2)

$\text{LHS} = \text{Sin}^2 A + \text{Sin}^2 B + \text{Sin}^2 C$

$= \frac{1 - \text{Cos } 2A}{2} + \frac{1 - \text{Cos } 2B}{2} + \frac{1 - \text{Cos } 2C}{2}$

$\text{Sin}^2 \theta = \frac{1 - \text{Cos } 2\theta}{2}$

$= \frac{1}{2}[1 - \text{Cos } 2A + 1 - \text{Cos } 2B + 1 - \text{Cos } 2C]$

$= \frac{1}{2}[3 - \{\text{Cos } 2A + \text{Cos } 2B + \text{Cos } 2C\}]$

$= \frac{1}{2}[3 - \{2 \text{ Cos } \left(\frac{2A + 2B}{2}\right) \text{Cos } \left(\frac{2A - 2B}{2}\right) + \text{Cos } 2C\}]$

$\text{Cos } C + \text{Cos } D = 2 \text{ Cos}\left(\frac{C + D}{2}\right) \text{ Cos } \left(\frac{C + D}{2}\right)$

$= \frac{1}{2}[3 - \{2\text{Cos } (A + B) \text{ Cos } (A - B) - \text{Cos } 2C\}]$

$= \frac{1}{2}[3 - \{- 2\text{Cos } C \text{ Cos } (A - B) + 2 \text{ Cos}^2 C - 1\}]$

$= \frac{1}{2}[3 - \{- 1 - 2 \text{ Cos } C \text{ (Cos } (A - B) - \text{Cos } C)\}]$

$= \frac{1}{2}[3 - \{-1 + 2 \text{ Cos } C \text{ (Cos } (A - B) + \text{Cos } (A + B))\}]$

$= \frac{1}{2}[3 - \{-1 - 2 \text{ Cos } C \text{ (2 Cos } A \text{ Cos } B)\}]$

$\text{Cos } (A + B) + \text{Cos } (A - B) = 2 \text{ Cos } A \text{ Cos } B$

$= \frac{1}{2}[3 - \{-1 - 4 \text{ Cos } A \text{ Cos } B \text{ Cos } C\}]$

$= \frac{1}{2}[3 + 1 + 4 \text{ Cos } A \text{ Cos } B \text{ Cos } C]$

$= \frac{1}{2}[4 + 4 \text{ Cos } A \text{ Cos } B \text{ Cos } C]$

$= \frac{4}{2}[1 + \text{Cos } A \text{ Cos } B \text{ Cos } C]$

$= 2[1 + \text{Cos } A \text{ Cos } B \text{ Cos } C] = \text{RHS}$

9. If $A + B + C = 180^0$, then prove that

$$Cos^2 A + Cos^2 B + Cos^2 C = 1 - 2Cos A Cos B Cos C)$$

Sol:

Given, $A + B + C = 180^0$

$A + B = 180^0 - C$

$Sin (A + B) = Sin (180^0 - C) = Sin C$

$Cos (A + B) = Cos (180^0 - C) = - Cos C$

$Sin (A + B) = Sin C$ ---------------- (1)

$Cos (A + B) = - Cos C$---------------- (2)

$LHS = Sin^2 A + Sin^2 B + Sin^2 C$

$= \frac{1+ Cos 2A}{2} + \frac{1+ Cos 2B}{2} + \frac{1+ Cos 2C}{2}$

$Cos^2 \theta = \frac{1+Cos 2\theta}{2}$

$= \frac{1}{2}[1 + Cos 2A + 1 + Cos 2B + 1 + Cos 2C]$

$= \frac{1}{2}[3 + \{Cos 2A + Cos 2B + Cos 2C\}]$

$= \frac{1}{2}[3+ \{2 Cos \left(\frac{2A + 2B}{2}\right) Cos \left(\frac{2A - 2B}{2}\right) + Cos 2C\}]$

$Cos C + Cos D = 2 Cos\left(\frac{C + D}{2}\right) Cos \left(\frac{C + D}{2}\right)$

$= \frac{1}{2}[3+ \{2Cos (A + B) Cos (A - B) - Cos 2C\}]$

$= \frac{1}{2}[3+ \{- 2Cos C Cos (A - B) + 2 Cos^2 C - 1\}]$

$= \frac{1}{2}[3+ \{- 1 - 2 Cos C (Cos (A - B) - Cos C)\}]$

$= \frac{1}{2}[3 + \{-1 + 2 Cos C (Cos (A - B) + Cos (A + B))\}]$

$= \frac{1}{2}[3 + \{-1-2 Cos C (2 Cos A Cos B)\}]$

$Cos (A + B) + Cos (A - B) = 2 Cos A Cos B$

$= \frac{1}{2}[3 + \{-1 - 4 Cos A Cos B Cos C\}]$

$= \frac{1}{2}[3 - 1 - 4 Cos A Cos B Cos C]$

$= \frac{1}{2}[2 - 4 Cos A Cos B Cos C]$

$= \frac{2}{2}[1 - 2Cos A Cos B Cos C]$

$=1 - 2Cos A Cos B Cos C] = RHS$

10. If $A + B + C = 2S$, then prove that

$$\text{Cos}(S - A) + \text{Cos}(S - B) + \text{Cos}(S - C) + \text{Cos}\, S = 4\,\text{Cos}\frac{A}{2}\,\text{Cos}\frac{B}{2}\,\text{Cos}\frac{C}{2}$$

Sol:

Given, $A + B + C = 2S$ ---------------- (1)

$\text{LHS} = \text{Cos}(S - A) + \text{Cos}(S - B) + \text{Cos}(S - C) + \text{Cos}\, S$

$$= 2\,\text{Cos}\left(\frac{S-A+S-B}{2}\right)\text{Cos}\left(\frac{S-A-S+B}{2}\right) + 2\,\text{Cos}\left(\frac{S-C+S}{2}\right)\text{Cos}\left(\frac{S-C-S}{2}\right)$$

$$= 2\,\text{Cos}\left(\frac{2S-A-B}{2}\right)\text{Cos}\left(\frac{B-A}{2}\right) + 2\,\text{Cos}\left(\frac{2S-C}{2}\right)\text{Cos}\left(\frac{-C}{2}\right)$$

$$= 2\,\text{Cos}\left(\frac{A+B+C-A-B}{2}\right)\text{Cos}\left(\frac{B-A}{2}\right) + 2\,\text{Cos}\left(\frac{2S-C}{2}\right)\text{Cos}\left(\frac{-C}{2}\right) \quad \text{[From (1)]}$$

$$= 2\,\text{Cos}\left(\frac{C}{2}\right)\text{Cos}\left(\frac{A-B}{2}\right) + 2\,\text{Cos}\left(\frac{2S-C}{2}\right)\text{Cos}\left(\frac{C}{2}\right)$$

$$= 2\,\text{Cos}\left(\frac{C}{2}\right)\left[\text{Cos}\left(\frac{A-B}{2}\right) + \text{Cos}\left(\frac{2S-C}{2}\right)\right]$$

$$= 2\,\text{Cos}\left(\frac{C}{2}\right)\left[2\text{Cos}\left(\frac{A-B+2S-C}{2}\right)\text{Cos}\left(\frac{A-B-2S+C}{2}\right)\right]$$

$$= 2\,\text{Cos}\left(\frac{C}{2}\right)\left[2\text{Cos}\left(\frac{A-B+A+B+C-C}{2}\right)\text{Cos}\left(\frac{A-B-(A+B+C)+C}{2}\right)\right] \quad \text{[From (1)]}$$

$$= 2\,\text{Cos}\left(\frac{C}{2}\right)\left[2\text{Cos}\left(\frac{2A}{2}\right)\text{Cos}\left(\frac{-2B}{2}\right)\right]$$

$$= 4\,\text{Cos}\frac{A}{2}\,\text{Cos}\frac{B}{2}\,\text{Cos}\frac{C}{2} = \text{RHS}$$