MODEL PAPER –I (2022)

TOTAL MARKS: 75
TIME: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. If $A = \{1, 2, 3, 4\}$ and $f: A \longrightarrow R$ is a function defined by defined by $f(x) = \frac{x^2 - x + 1}{x + 1}$, then find range of f.
2. Find the domain of the real valued function $f(x) = \frac{\sqrt{3+x} + \sqrt{3-x}}{x}$.
3. If $A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ and $A + B = X$, then find x_1, x_2, x_3 and x_4.
4. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & x & 7 \end{bmatrix}$ is a symmetric matrix, then find x.
5. If $A = \begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$, $a^2 + b^2 + c^2 + d^2 = 1$, then find the inverse of A.
6. If $A = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$, then find – 5A.
7. Find the unit vector in the direction of the sum of the vectors $\bar{a} = 2\bar{i} + 2\bar{j} - 5\bar{k}$ and $\bar{b} = 2\bar{i} + \bar{j} + 3\bar{k}$.
8. Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$.
9. ABCDE is a pentagon. If the sum of **AB**, **AE**, **BC**, **DC**, **ED** and **AC** is λ **AC**, then find the value of λ.
10. Find the Cartesian equation of the plane passing through the point (– 2, 1, 3) and perpendicular to the vector $3\bar{i} + \bar{j} + 5\bar{k}$.
11. If $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$ and $\bar{b} = 3\bar{i} + 5\bar{j} - \bar{k}$ are two sides of a triangle, then find its area.
12. If $\sin\alpha + \operatorname{cosec}\alpha = 2$, find the value of $\sin^n\alpha + \operatorname{cosec}^n\alpha$, $n \in Z$.
13. Prove that $\frac{\cos 9^0 + \sin 9^0}{\cos 9^0 - \sin 9^0} = \cot 36^0$.
14. If $\sinh x = 3$, then show that $x = \log_e(3 + \sqrt{10})$.
15. Prove that $(\cosh x - \sinh x)^n = \cosh nx - \sinh nx$ for any $n \in R$.

II. Short answer type questions $5 \times 4 = 20$

16. Show that $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find 3B – 2A.
17. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ then fin $A^3 - 3A^2 - A - 3I$, where I is unit matrix of order 3..
18. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then find $(A')^{-1}$.
19. Is the triangle formed by the vectors $3\bar{i} + 5\bar{j} + 2\bar{k}$, $2\bar{i} - 3\bar{j} - 5\bar{k}$ and $-5\bar{i} - 2\bar{j} + 3\bar{k}$ equilateral?
20. $\bar{a}$, $\bar{b}$, $\bar{c}$ are non-coplanar vectors, prove that the points $-\bar{a} + 4\bar{b} - 3\bar{c}$, $3\bar{a} + 2\bar{b} - 5\bar{c}$, $-3\bar{a} + 8\bar{b} - 5\bar{c}$ and $-3\bar{a} + 2\bar{b} + \bar{c}$ are coplanar.

21. If $|\bar{a}| = 2$, $|\bar{b}| = 3$ and $|\bar{c}| = 4$ and each $\bar{a}, \bar{b}, \bar{c}$ is perpendicular to the sum of the other two vectors, then find the magnitude of $\bar{a} + \bar{b} + \bar{c}$.
22. Let $\bar{a} = 4\bar{i} + 5\bar{j} - \bar{k}$, $\bar{b} = \bar{i} - 4\bar{j} + 5\bar{k}$ and $\bar{c} = 3\bar{i} + \bar{j} - \bar{k}$. Find the vector $\bar{\alpha}$ which is perpendicular to both $\bar{a}$ and $\bar{b}$ and $\bar{\alpha} \cdot \bar{c} = 21$.
23. Prove that $(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$.
24. If A is not an integral multiple of $\frac{\pi}{2}$, prove that (i) $\tan A + \cot A = 2 \operatorname{cosec} 2A$ (ii) $\cot A - \tan A = 2 \cot 2A$.
25. If $\theta \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$ and $x = \log_e\left(\cot\left(\frac{\pi}{4} + \theta\right)\right)$, then prove that $\cosh x = \sec 2\theta$.
26. In Δ ABC, prove that $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \frac{s^2}{\Delta}$.
27. In Δ ABC, show that $\sum\left(\frac{r_1}{(s-b)(s-c)}\right) = \frac{3}{r}$.

III.Long answer type questions **5 × 7 = 35**

28. If $f = \{(4, 5), (5, 6), (6, -4)\}$ and $g = \{(4, -4), (6, 5), (8, 5)\}$, then find (i) $f + g$ (ii) $f - g$ (iii) $2f + 4g$ (iv) f/g (v) fg (vi) $|f|$ (vii) $\sqrt{f}$.
29. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then show that the adjoint of A is 3A'. Find A^{-1}.
30. By using **Matrix inversion method**, solve the following system of equations. $2x - y + 3z = 8$, $-x + 2y + z = 4$ and $3x + y - 4z = 0$.
31. Solve the following equation by using **Cramer's rule** $x + y + z = 9$, $2x + 5y + 7z = 52$ and $2x + y - z = 0$.
32. If **i**, **j**, **k** are unit vectors along the positive direction of the coordinate axes, then show that the four points 4**i** + 5**j** + **k**, – **j** – **k**, 3**i** + 9**j** + 4**k** and – 4**i** + 4**j** + 4**k** are coplanar.
33. If **a**, **b**, **c** are unit vectors such that **a** is perpendicular to the plane of **b**, **c** and the angle between b and c is $\frac{\pi}{3}$, then find |**a** + **b** + **c**|
34. If [**b c d**] + [**c a d**] + [**a b d**] = [**a b c**], then show that the points with position vectors **a**, **b**, **c** and **d** are coplanar.
35. If A, B, C are the angles of a triangle, then prove that $\sin A + \sin B - \sin C = 4 \sin\frac{A}{2} \sin\frac{B}{2} \cos\frac{C}{2}$.
36. Show that $\frac{1}{r^2} + \frac{1}{{r_1}^2} + \frac{1}{{r_2}^2} + \frac{1}{{r_3}^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$
37. Show that $\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2 + \frac{r}{2R}$

❁ ❁ ❁