MODEL PAPER –II (2022)

TOTAL MARKS: 75
TIME: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. If A = {– 2, – 1, 0, 1, 2} and f: A ⟶ B is a surjection defined by $f(x) = x^2 + x + 1$, then find B.
2. Find the domain and range of the real valued function $f(x) = \sqrt{9 - x^2}$.
3. If $A = \begin{bmatrix} 2 & 3 & -1 \\ 7 & 8 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -4 & -1 \end{bmatrix}$ then find A + B.
4. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = 0$, then find the value of k.
5. If $A = \begin{bmatrix} 2 & -4 \\ -5 & 3 \end{bmatrix}$, then find A + A' and AA'.
6. Find A^2 when $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$.
7. Let $\bar{a} = \bar{i} + 2\bar{j} + 3\bar{k}$ and $\bar{b} = 3\bar{i} + \bar{j}$. Find the unit vector in the direction of $\bar{a} + \bar{b}$.
8. If $\bar{a}$, $\bar{b}$, $\bar{c}$ are the position vectors of the vertices A, B, C respectively of Δ ABC, then find the vector equation of the median through the vertex A.
9. If the vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}$ and $\mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear, then find λ and μ.
10. Let $\bar{e_1}$ and $\bar{e_2}$ be unit vectors making an angle θ. If $\frac{1}{2}|\bar{e_1} - \bar{e_2}| = \sin \lambda\theta$, then find λ.
11. Find the unit vector perpendicular to both $\bar{i} + \bar{j} + \bar{k}$ and $2\bar{i} + \bar{j} + 3\bar{k}$
12. Eliminate θ: $x = a \cos^3 \theta$, $y = b \sin^3\theta$.
13. Find the period of the function $f(x) = \cos\left(\frac{4x + 9}{5}\right)$.
14. If $\cosh x = \sec \theta$, then prove that $\tanh^2\left(\frac{x}{2}\right) = \tan^2\left(\frac{\theta}{2}\right)$.
15. For x ∈ R, prove that $\cosh^4 x - \sinh^4 x = \cosh 2x$.

II. Short answer type questions $5 \times 4 = 20$

16. Show that $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$, then find AB and BA.
17. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then show that $A^{-1} = A^3$.
18. If $\theta - \emptyset = \frac{\pi}{2}$, then show that $\begin{bmatrix} \cos^2\theta & \cos\theta \sin\theta \\ \cos\theta \sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\emptyset & \cos\emptyset \sin\emptyset \\ \cos\emptyset \sin\emptyset & \sin^2\emptyset \end{bmatrix} = 0$.
19. Show that the points $A(2\bar{i} - \bar{j} + \bar{k})$, $B(\bar{i} - 3\bar{j} - 5\bar{k})$, $C(3\bar{i} - 4\bar{j} - 4\bar{k})$ are the vertices of a right-angled triangle.
20. Show that the line joining the pair of points $6\bar{a} - 4\bar{b} + 4\bar{c}$, $-4\bar{c}$ and the line joining the pair of points $-\bar{a} - 2\bar{b} - 3\bar{c}$, $\bar{a} + 2\bar{b} - 5\bar{c}$ intersects at the point $-4\bar{c}$, when $\bar{a}$, $\bar{b}$, $\bar{c}$ are non-coplanar vectors.

21. Prove that the angle θ between any two diagonals of a cube is given by $\cos\theta = \frac{1}{3}$.

22. Let $\bar{a} = 2\bar{i} + \bar{j} - 2\bar{k}$ and $\bar{b} = \bar{i} + \bar{j}$. If $\bar{c}$ is a vector such that $\bar{a}.\bar{c} = |\bar{c}|, |\bar{c} - \bar{a}| = 2\sqrt{2}$ and angle between $\bar{a} \times \bar{b}$ and $\bar{c}$ is 30^0 then find the value of $|(\bar{a} \times \bar{b}) \times \bar{c}|$.

23. If $\tan 20^0 = p$, then prove that $\frac{\tan 610^0 + \tan 700^0}{\tan 560^0 - \tan 470^0} = \frac{1-p^2}{1+p^2}$.

24. Prove that $\cos^2\theta + \cos^2\left(\frac{2\pi}{3} + \theta\right) + \cos^2\left(\frac{2\pi}{3} - \theta\right) = \frac{3}{2}$.

25. Prove that $\frac{\cosh x}{1-\tanh x} + \frac{\sinh x}{1-\coth x} = \sinh x + \cosh x$

26. In Δ ABC, prove that $\frac{a}{bc} + \frac{\cos A}{a} = \frac{b}{ca} + \frac{\cos B}{b} = \frac{c}{ab} + \frac{\cos C}{c}$.

27. In Δ ABC, show that $r + r_1 + r_2 - r_3 = 4R\cos C$.

III.Long answer type questions **5 × 7 = 35**

28. If the function f is defined by $f(x) = \begin{cases} 3x - 2, & x > 3 \\ x^2 - 2, & -2 \le x \le 2 \\ 2x + 1, & x < -2 \end{cases}$, then find the values, if exist, of f(4), f(2.5), f(– 2.5), f(– 2), f(– 4), f(0) and f (– 7)

29. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix, then prove that A is invertible and $A^{-1} = \frac{\text{adj } A}{\det A}$.

30. By using **Matrix inversion method**, solve the following system of equations.
$2x - y + 3z = 9$, $x + y + z = 6$ and $x - y + z = 2$.

31. Solve the following equation by using **Cramer's rule**
$5x - 6y + 4z = 15$, $7x + 4y - 3z = 19$ and $2x + y + 6z = 46$.

32. If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $\bar{c} = -\bar{i} + \bar{j} + 2\bar{k}$, and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$.

33. If $\bar{a} = \bar{i} + \bar{j} + \bar{k}$, $\bar{c} = \bar{j} - \bar{k}$, then find vector $\bar{b}$ such that $\bar{a} \times \bar{b} = \bar{c}$ and $\bar{a}.\bar{b} = 3$.

34. If $\bar{a} = (1, -1, -6)$, $\bar{b} = (1, -3, 4)$ and $\bar{c} = (2, -5, 3)$, then compute the following
(i) $\bar{a}.(\bar{b} \times \bar{c})$ (ii) $\bar{a} \times (\bar{b} \times \bar{c})$ (iii) $(\bar{a} \times \bar{b}) \times \bar{c}$

35. If A, B, C are the angles of a triangle, then prove that
$\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$.

36. If $\cot\frac{A}{2} : \cot\frac{B}{2} : \cot\frac{C}{2} = 3 : 5 : 7$, then show that a : b : c = 6 : 5 : 4.

37. Show that $\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{r} - \frac{1}{2R}$

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