MODEL PAPER –III (2022)

TOTAL MARKS: 75
TIME: 3hrs.

I. Very short answer type questions **$10 \times 2 = 20$**

1. If $f: R - \{0\} \longrightarrow R$ is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$.
2. Find the domain and range of the function $f(x) = \frac{x}{2-3x}$.
3. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, then find A^2.
4. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then show that $AA' = A'A = I$.
5. Find the adjoint and inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$.
6. Construct a 3×2 matrix whose elements are defined by $a_{ij} = \frac{1}{2}|i - 3j|$
7. If the position vectors of the points A, B and C are $-2\bar{i} + \bar{j} - \bar{k}$, $-4\bar{i} + 2\bar{j} + 2\bar{k}$ and $6\bar{i} - 3\bar{j} - 13\bar{k}$ respectively and $\mathbf{AB} = \lambda\,\mathbf{AC}$, then find λ.
8. Find the vector equation of the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $-4\bar{i} + 3\bar{j} - \bar{k}$.
9. Write the direction ratios of the vector $\bar{a} = \bar{i} + \bar{j} - 2\bar{k}$ and hence calculate its direction cosines.
10. If $\bar{a} = 6\bar{i} + 2\bar{j} + 3\bar{k}$ and $\bar{b} = 2\bar{i} - 9\bar{j} + 6\bar{k}$, then find **a . b** and find the angle between **a** and **b.**
11. If $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$, then show that **a + b** and **a – b** are perpendicular to each other.
12. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.
13. Find the cosine function whose period is 7.
14. If $\cos hx = \frac{5}{2}$, then find cosh (2x) and sinh (2x).
15. Show that $\text{Tanh}^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e 3$.

II. Short answer type questions **$5 \times 4 = 20$**

16. Show that $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ is non – singular matrix and find A^{-1}.
17. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then show that $(aI + bE)^3 = a^3I + 3a^2bE$.
18. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 4 \\ 4 & 5 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then find B – A and 4A – 5B.
19. In Δ ABC, if O is the circumcentre and H is the orthocentre, then show that
    (i) $\overline{OA} + \overline{OB} + \overline{OC} = \overline{OH}$ (ii) $\overline{HA} + \overline{HB} + \overline{HC} = 2\overline{HO}$
20. In the two-dimensional plane, prove by vector methods, the equation of the line whose intercepts on the axes are a and b is $\frac{x}{a} + \frac{y}{b} = 1$.

21. The vectors $\mathbf{AB} = 3\bar{i} - 2\bar{j} + 2\bar{k}$ and $\mathbf{AD} = \bar{i} - 2\bar{k}$ represents the adjacent sides of a parallelogram ABCD. Find the angle between the diagonals.

22. Find a unit vector perpendicular to the plane determined by the points P (1, – 1, 2), Q (2, 0, – 1) and R (0, 2, 1).

23. Find the range of the function $f(x) = 7\cos x - 24\sin x + 5$.

24. If $\cos x + \cos y = \frac{4}{5}$ and $\cos x - \cos y = \frac{2}{7}$, then find the value of $14\tan\left(\frac{x-y}{2}\right) + 5\cot\left(\frac{x+y}{2}\right)$.

25. If $u = \log_e\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$ and $\cos\theta > 0$, then prove that $\cosh u = \sec\theta$.

26. In Δ ABC, if $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, show that $C = 60^0$

27. In Δ ABC, show that $r + r_3 + r_1 - r_2 = 4R\cos B$.

III.Long answer type questions $5 \times 7 = 35$

28. If f = {(1, 2), (2, –3), (3, –1)}, then find (i) 2f (ii) 2 + f (iii) f^2 (iv) $\sqrt{f}$

29. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that $A^2 - 4A - 5I = 0$.

30. By using **Matrix inversion method**, solve the following system of equations.
$x + y + z = 9$, $2x + 5y + 7z = 52$ and $2x + y - z = 0$.

31. Solve the following equation by using **Cramer's rule**
$3x + 4y + 5z = 18$, $2x - y + 8z = 13$ and $5x - 2y + 7z = 20$.

32. Find the point of intersection of the lines $\bar{r} = 2\bar{a} + \bar{b} + t(\bar{b} - \bar{c})$ and $\bar{r} = \bar{a} + x(\bar{b} + \bar{c}) + y(\bar{a} + 2\bar{b} - \bar{c})$ where $\bar{a}, \bar{b}$ and $\bar{c}$ are non – coplanar vectors.

33. G is the centroid of the Δ ABC and a, b, c are the lengths of the sides BC, CA and retrospectively, then prove that $a^2 + b^2 + c^2 = 3(\mathbf{OA}^2 + \mathbf{OB}^2 + \mathbf{OC}^2) - 9(\mathbf{OG})^2$

34. If $\bar{a} = 3\bar{i} - \bar{j} + 2\bar{k}$, $\bar{b} = -\bar{i} + 3\bar{j} + 2\bar{k}$, $\bar{c} = 4\bar{i} + 5\bar{j} - 2\bar{k}$, and $\bar{d} = \bar{i} + 3\bar{j} + 5\bar{k}$, then compute
(i) $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})$ (ii) $(\bar{a} \times \bar{b}).\bar{c} - (\bar{a} \times \bar{d}).\bar{b}$

35. If A + B + C = 0, then prove that $\cos^2 A + \cos^2 B + \cos^2 C = 1 + 2\cos A\cos B\cos C$.

36. If $a^2 + b^2 + c^2 = 8R^2$, then prove that the triangle is a right triangle.

37. If a = 13, b = 14, c = 15, then show that $R = \frac{65}{8}$, $r = 4$, $r_1 = \frac{21}{2}$, $r_2 = 12$ and $r_3 = 14$.

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