MODEL PAPER –IV (2022)

TOTAL MARKS: 75
TIME: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. If $A = \{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\}$ and f: A $\longrightarrow$ B is a surjection defined by $f(x) = \cos x$, then find B.
2. Find the domain of the function $\frac{1}{(x^2-1)(x+3)}$.
3. If $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$ then find the values of x, y, z and a.
4. Find the trace of the matrix $\begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 1 & 0 & 1 \end{bmatrix}$.
5. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, then show that $A^2 = -I$.
6. If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew symmetric matrix, then find the value of x.
7. Find unit vector in the direction of sum of the vectors $\bar{a} = 2\bar{i} + 2\bar{j} - 5\bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} + 3\bar{k}$.
8. If $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors, then find the values of m and n.
9. OABC is a parallelogram. If $\overline{OA} = \bar{a}$ and $\overline{OC} = \bar{c}$ then, find the vector equation of BC.
10. Find the angle between the vectors $\bar{i} + 2\bar{j} + 3\bar{k}$ and $3\bar{i} - \bar{j} + 2\bar{k}$
11. If $\bar{a} = 2\bar{i} - 3\bar{j} + 5\bar{k}$ and $\bar{b} = -\bar{i} + 4\bar{j} + 2\bar{k}$, then find $\bar{a} \times \bar{b}$ and find unit vector perpendicular to both $\bar{a}$ and $\bar{b}$.
12. Evaluate $\cos^2 45^0 + \cos^2 135^0 + \cos^2 225^0 + \cos^2 315^0$
13. Find the period of the function $f(x) = \tan(x + 4x + 9x + \ldots\ldots + n^2x)$.
14. If $\sin hx = \frac{3}{4}$, then find cosh (2x) and sinh (2x).
15. If sin hx =5, then show that $x = \log_e(5 + \sqrt{26})$.

II. Short answer type questions $5 \times 4 = 20$

16. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, then examine whether A and B commute with respect to multiplication of matrices.
17. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then find A^4.
18. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then show that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for any integer $n \geq 1$, by using mathematical induction method.
19. If ABCDEF is a regular hexagon with center 'O', then show that **AB +AC + AD +AE +AF = 3AD = 6 OA.**
20. Find the equation of the line parallel to the vector $2\bar{i} - \bar{j} + 2\bar{k}$ and which passes through the point A whose position vector is $3\bar{i} + \bar{j} - \bar{k}$. If P is a point on this line such that AP = 15, find the position vector of P.

21. If $2\bar{i} + \lambda\bar{j} - \bar{k}$ and $4\bar{i} - 2\bar{j} + 2\bar{k}$ are perpendicular to each other, then find λ.
22. Let $\bar{a}$ and $\bar{b}$ be vectors satisfying $|\bar{a}| = |\bar{b}| = 5$and $(\bar{a}, \bar{b}) = 45^0$. Find the area of the triangle having $\bar{a} - 2\bar{b}$ and $3\bar{a} + 2\bar{b}$ as two of its sides.
23. If $\frac{\sin\alpha}{a} = \frac{\cos\alpha}{b}$ then prove that $a \sin 2\alpha + b \cos 2\alpha = b$.
24. If $A + B = \frac{\pi}{4}$, then prove that (i) $(1 + \tan A)(1 + \tan B) = 2$ (ii) $(\cot A - 1)(\cot B - 1) = 2$
25. Prove that $\frac{\tanh x}{\operatorname{sech} x - 1} + \frac{\tan hx}{\operatorname{sech} x + 1} = -2 \operatorname{cosech} x$
26. If $\sin\theta = \frac{a}{b+c}$, then prove that $\cos\theta = \frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$.
27. In Δ ABC, if $(r_2 - r_1)(r_3 - r_1) = 2r_2 r_3$, then show that $A = 90^0$.

III.Long answer type questions **$5 \times 7 = 35$**

28. If f and g are real valued functions defined by $f(x) = 2x - 1$ and $g(x) = x^2$ then find
    (i) $(3f - 2g)(x)$ (ii) $(fg)(x)$ (iii) $\left(\frac{\sqrt{f}}{g}\right)(x)$ (iv) $(f + g + 2)(x)$
29. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$, then show that $A^{-1} = A'$.
30. By using **Matrix inversion method**, solve the following system of equations.
    $3x + 4y + 5z = 18$, $2x - y + 8z = 13$ and $5x - 2y + 7z = 20$.
31. Solve the following equation by using **Cramer's rule**
    $x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$.
32. Find the vector equation of the plane passing through the points $4\bar{i} - 3\bar{j} - \bar{k}$, $3\bar{i} + 7\bar{j} - 10\bar{k}$, $2\bar{i} + 5\bar{j} - 7\bar{k}$ and show that the point $\bar{i} + 2\bar{j} - 3\bar{k}$ lie on the plane.
33. A line makes an angles θ_1, θ_2, θ_3 and θ_4 with the diagonals of a cube. Show that
    $\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 + \cos^2\theta_4 = \frac{4}{3}$.
34. If $\bar{a} = 7\bar{i} - 2\bar{j} + 3\bar{k}$, $\bar{b} = 2\bar{i} + 8\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$, then compute $\bar{a} \times \bar{b}$, $\bar{a} \times \bar{c}$ and $\bar{a} \times (\bar{b} + \bar{c})$. Verify whether the cross product is distributive over vector addition.
35. In triangle ABC prove that
    $\operatorname{Cos}\frac{A}{2} + \operatorname{Cos}\frac{B}{2} - \operatorname{Cos}\frac{C}{2} = 4\cos\left(\frac{\pi + A}{4}\right)\cos\left(\frac{\pi + B}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$
36. If $\operatorname{Cot}\frac{A}{2} : \operatorname{Cot}\frac{B}{2} : \operatorname{Cot}\frac{C}{2} = 3 : 5 : 7$, then show that $a : b : c = 6 : 5 : 4$.
37. In Δ ABC, show that $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$.