MODEL PAPER – I

TOTAL MARKS: 75
Time: 3hrs.

I. Very short answer type questions **$10 \times 2 = 20$**

1. Find the value of y, if the line joining the points (3, y) and (2, 7) is parallel to the line joining the points (– 1, 4) and (0, 6).
2. Transform the equation $\frac{x}{a} + \frac{y}{b} = 1$ into the normal form when a> 0 and b > 0. If the perpendicular distance of the straight line from the origin is p, deduce that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.
3. Find the ratio in which the straight line $2x + 3y = 5$ divides the line segment joining the points (0, 0) and (– 2, 1). State whether the points lie on the same side or on either side of the straight-line.
4. Find the value of k, if the straight lines $y - 3kx + 4 = 0$ and $(2k - 1)x - (8k - 1)y - 6 = 0$ are perpendicular.
5. Show that the points (1, 2, 3), (2, 3, 1) and (3, 1, 2) form an equilateral triangle.
6. Find the fourth vertex of the parallelogram whose consecutive vertices are (2, 4, – 1), (3, 6, – 1) and (4, 5, 1).
7. If the point (1, 2, 3) is changed to the point (2, 3, 1) through translation of axes. Find the new origin.
8. Find the equation of the plane passing through (– 2, 1, 3) and having (3, – 5, 4) are dr's of its normal.
9. Sow that $\lim_{x\to 0^+}\left(\frac{2\,|x|}{x} + x + 1\right) = 3$.
10. Compute $\lim_{x\to 0} \frac{e^x - \sin x - 1}{x}$.
11. Find $\lim_{x\to\infty}\left(\sqrt{x+1} - \sqrt{x}\right)$.
12. Find the derivative of the function $f(x) = 5\sin x + e^x \log x$.
13. Find the derivative of the function $f(x) = \sin(\cos(x^2))$.
14. Find dy and Δy of $y = f(x) = 5x^2 + 6x + 6$ at $x = 2$ when $\Delta x = 0.001$.
15. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x coordinate is 2.

II. Short answer type questions **$5 \times 4 = 20$**

16. A (2, 3) and B (– 3, 4) are two given points. Find the equation of locus of P so that the area of the triangle PAB is 8.5.
17. Find the equation of locus of P, if the line segment joining (2, 3) and (– 1, 5) subtends a right angle at P.
18. Find the equation of locus of a point, the sum of whose distance from (0, 2) and (0, −2) is 6.
19. When the origin is shifted to (−1, 2) by the translation of axes find the transformed equation of the curve $x^2 + y^2 + 2x - 4y + 1 = 0$.

20. When the axes are rotated through an angle 45^0, the transformed equation of a curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.
21. Find the value of k, if the lines $2x - 3y + k = 0$, $3x - 4y - 13 = 0$ and $8x - 11y - 33 = 0$ are concurrent.
22. A straight line through P (3, 4) makes an angle 60^0 with the positive direction of the X – axis. Find the coordinates of the points on the line which are 5 units away from P.
23. Show that the points O (0, 0, 0), A (2, −3, 3) and B (−2, 3, −3) are collinear. Find the ratio in which each point divides the segment joining the other two.
24. Compute $\lim_{x \to \infty} \frac{x^2 - \sin x}{x^2 - 2}$.
25. If $x^3 + y^3 - 3axy = 0$, then find $\frac{dy}{dx}$.
26. If the increase in the radius of a square is 2% then find the approximate percentage of increase in its area.
27. Show that the tangent at ani point θ on the curve $x = c \sec \theta$, $y = c \tan \theta$ is $y \sin \theta = x - c \cos \theta$.

III. Long answer type questions **$5 \times 7 = 35$**

28. Find the orthocentre of the triangle formed by the lines $x + 2y = 0$, $4x + 3y - 5 = 0$ and $3x + y = 0$.
29. Find the circumcentre of the triangle whose vertices are (1, 3), (−3, 5) and (5, −1)
30. If Q (h, k) is the image of the point P (x_1, y_1) with respect to the straight line $ax + by + c = 0$, then prove that $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$. Hence find the image of (1, −2) with respect to the straight line $2x - 3y + 5 = 0$.
31. Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is $\left|\frac{n^2\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}\right|$.
32. Find the value of k, if the line joining the origin to the point of intersection of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.
33. Find the direction cosines of two lines which are connected by the relations $l + m + n = 0$ and $mn - 2nl - 2lm = 0$.
34. Find the derivative of the function $f(x) = \frac{x(1 + x^2)}{\sqrt{1 - x^2}}$.
35. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then show that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$.
36. If the tangent at any point P on the curve $x^m y^n = a^{m+n}$ $(mn \neq 0)$ meets the coordinate axes in A, B, then show that AP : BP is a constant.
37. From a rectangular sheet of dimensions 30 cm × 80 cm, four equal squares of side x cm are removed at the corners, and the sides are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of box is the greatest.

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