

## MODEL PAPER - III (2022)

TOTAL MARKS: 75

Time: 3hrs.

## I. Very short answer type questions

10 × 2 = 20

1. Find the equation of the straight line passing through A (-1, 3) and (i) parallel (ii) perpendicular to the straight line passing through B(2, -5) and C (4, 6).
2. If the area of the triangle formed by the straight lines  $x = 0$ ,  $y = 0$  and  $3x + 4y = a$  ( $a > 0$ ) is 6, find the value of  $a$ .
3. Transform the equation  $x + y + 1 = 0$  into normal form.
4. Find the value of  $p$ , if the straight lines  $3x + 4y = 5$ ,  $2x + 3y = 4$  and  $px + 4y = 6$  are concurrent.
5. Find  $x$  if the distance between (5, -1, 7) and (x, 5, 1) is 9 units.
6. Find the centroid of the triangle whose vertices are (5, 4, 6), (1, -1, 3) and (4, 3, 2).
7. Find the ratio in which the point P (5, 4, -6) divides the line segment joining the points A (3, 2, -4) and B (9, 8, -10). Also find the harmonic conjugate of P.
8. Find the angle between the planes  $x + 2y + 2z - 5 = 0$  and  $3x + 3y + 2z - 8 = 0$ .
9. compute  $\lim_{x \rightarrow 2} \left( \frac{x^2 - 5}{4x + 10} \right)$ .
10. Compute  $\lim_{x \rightarrow 2^+} ([x] + x)$  and  $\lim_{x \rightarrow 2^-} ([x] + x)$
11. Find  $\lim_{x \rightarrow 0} \frac{a^x - 1}{b^x - 1}$ .
12. If  $f(x) = x \cdot e^x \cdot \sin x$ , then find  $f'(x)$ .
13. If  $y = \sin^{-1} \sqrt{x}$ , then find  $\frac{dy}{dx}$ .
14. Find  $dy$  and  $\Delta y$  of  $y = f(x) = x^2 + x$  at  $x = 10$  when  $\Delta x = 0.1$ .
15. Find the slope of the tangent to the curve  $y = 5x^2$  at (-1, 5).

## II. Short answer type questions

5 × 4 = 20

16. Find the equation of the locus of a point which is at a distance 5 from A (4, -3).
17. Find the equation of locus of a point which is equidistance from the coordinate axes.
18. A (1, 2), B (2, -3) and C (-2, 3) are three points. A point P moves such that  $PA^2 + PB^2 = 2 PC^2$ .  
Show that the equation of the locus of P is  $7x - 7y + 4 = 0$ .
19. When the origin is shifted to the point (-1, 2) by the translation of axes find the transformed equation of the curve  $2x^2 + y^2 - 4x + 4y = 0$ .
20. When the axes are rotated through an angle  $\frac{\pi}{4}$ , find the transformed equation of  $3x^2 + 10xy + 3y^2 = 9$ .

21. A straight line parallel to the line  $y = \sqrt{3}x$  passes through Q (2, 3) and cuts the line  $2x + 4y - 27 = 0$  at P. find the length of PQ.
22. A triangle of area 24 sq. units is formed by a straight line and the coordinate axes in the first quadrant. Find the equation of the straight line passes through (3, 4).
23. Show that the (3, 2, - 4), B (5, 4, - 6) and C (9, 8, - 10) are collinear and find the ratio in which B divides AC.
24. Compute  $\lim_{x \rightarrow 3} \frac{x^2 - 8x + 15}{x^2 - 9}$ .
25. Find the derivative of the function  $f(x) = \sin 2x$  from the first principle of derivative.
26. If the increase in the side of a square is 4%, then find the approximate percentage of increase in the area of the square.
27. Show that at any point (x, y) on the curve  $y = b e^{\frac{x}{a}}$  the length of subtangent is constant and length of subnormal is  $\frac{y^2}{a}$ .

### III. Long answer type questions

5 × 7 = 35

28. Find the equation of the straight line passing through the point of intersection of the straight lines  $3x + 2y = 4$ ,  $2x + 5y = 1$  and whose distance from (2, - 1) is 2.
29. Find the circumcentre of the triangle whose vertices are (-2, 3), (2, -1) and (4, 0).
30. Find the orthocentre of the triangle formed by the lines  $x + 2y = 0$ ,  $4x + 3y - 5 = 0$  and  $3x + y = 0$ .
31. Show that the product of the perpendicular distances from a point  $(\alpha, \beta)$  to the pair of straight lines  $ax^2 + 2hxy + by^2 = 0$  is  $\frac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(a-b)^2 + 4h^2}}$ .
32. Show that the lines joining the origin to the points of intersection of the curve  $x^2 - xy + y^2 + 3x + 3y - 2 = 0$  and the straight-line  $x - y - \sqrt{2} = 0$  are mutually perpendicular.
33. Find the direction cosines of two lines which are connected by the relation  $l - 5m + 3n = 0$  and  $7l^2 + 5m^2 - 3n^2 = 0$ .
34. If  $y = x\sqrt{a^2 + x^2} + a^2 \log(x + \sqrt{a^2 + x^2})$ , then prove that  $\frac{dy}{dx} = 2\sqrt{a^2 + x^2}$
35. Find the derivative of the function  $f(x) = \frac{x(1+x^2)}{\sqrt{1-x^2}}$ .
36. If the tangent at any point on the curve  $x^{2/3} + y^{2/3} = a^{2/3}$  intersects the coordinate axes in A and B, then show that the length AB is constant.
37. A window is in the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 20 feet. find the maximum area.

