MODEL PAPER – IV (2022)

TOTAL MARKS: 75
Time: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. Find the equation of the straight line passing through the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$.
2. Find the straight line which makes 150^0 with the positive direction of X – axis in the positive direction and passing through the point $(-2, -1)$.
3. If the product of intercepts made by the straight-line $x \tan\alpha + y \sec\alpha = 1$ $(0 \le \alpha < \frac{\pi}{2})$ on the coordinate axes is equal to $\sin\alpha$. Find α.
4. If $3a + 2b + 4c = 0$, then show that the equation $ax + by + c = 0$ represents a family of concurrent straight – lines and find the point of contact.
5. Show that the points (1, 2, 3), (7, 0, 1) and (−2, 3, 4) are collinear.
6. Find the centroid of the tetrahedron whose vertices are (2, 3, −4), (−3, 3, −2), (−1, 4, 2) and (4, 3, 2).
7. Find the ratio in which the XZ - plane divides the line joining A (−2, 3, 4) and B (1, 2, 3).
8. Find the equation of the plane if the foot of the perpendicular from origin to the plane is (1, 3, – 5).
9. compute $\lim_{x\to 2}\left(\frac{x-2}{x^3-8}\right)$.
10. Compute $\lim_{x\to 0}\left(\frac{\sqrt{1+x}-1}{x}\right)$.
11. Find $\lim_{x\to\infty}\frac{8|x|+3x}{3|x|-2x}$.
12. If $f(x) = 1 + x + x^2 + ... x^{100}$ then find $f'(1)$.
13. Find the derivative of the function $f(x) = Tan^{-1}(\log x)$.
14. Find dy and Δy of $y = f(x) = x^2 + 3x + 6$ at $x = 10$ when $\Delta x = 0.01$.
15. Find the slope of the normal to the curve $x = a\cos^3\theta$ and $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

II. Short answer type questions $5 \times 4 = 20$

16. Find the equation of the locus of a point P such that $PA^2 + PB^2 = 2c^2$, where A = (a, 0), B = (– a, 0).
17. Find the equation of locus of a point, the difference of whose distance from (– 5, 0) and (5, 0) is 8.
18. Find the equation of locus of P, if the distance of P from A (3, 0) is twice the distance of P form B (−3, 0).
19. When the origin is shifted to (3, 4) by the translation of axes find the transformed equation of the curve $2x^2 + 4xy + 5y^2 = 0$.
20. When the axes are rotated through an angleα, find the transformed equation of $x\cos\alpha + y\sin\alpha = p$

21. Find the equation of the straight line passing through the point $(-3, 2)$ and making an angle of 45^0 with the straight line $3x - y + 4 = 0$.

22. Transform the equation $\sqrt{3}\,x + y + 10 = 0$ into (i) slope intercept form (ii) intercept form and (iii) normal form.

23. If H, G, S and I respectively denote orthocentre, centroid, circumcentre and incentre of a triangle formed by the points (1, 2, 3), (2, 3, 1) and (3, 1, 2), then find H, G, S and I.

24. Compute $\lim_{x \to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x}$.

25. If $y = e^{a \sin^{-1} x}$ then prove that $\frac{dy}{dx} = \frac{ay}{\sqrt{1 - x^2}}$.

26. The radius of a sphere is measured as 14 cm. Later it was found the there is an error 0.02 cm in measuring the radius. Find the approximate error in surface area of the sphere.

27. Find the equations of the tangent and normal to the curve $y = 2\,e^{-\frac{x}{3}}$ at the point where the curve meets the Y – axis.

III. Long answer type questions **$5 \times 7 = 35$**

28. If Q (h, k) is the foot of the perpendicular from P (x_1, y_1) on the straight line $ax + by + c = 0$, then prove that $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(a x_1 + b y_1 + c)}{a^2 + b^2}$, hence find the foot of the perpendicular from $(-1, 3)$ on the straight line $5x - y - 18 = 0$.

29. If the equations of the sides of a triangle are $7x + y - 10 = 0$ and $x - 2y + 5 = 0$ and $x + y + 2 = 0$, find the orthocentre of the triangle.

30. If p and q are the lengths of the perpendicular from the origin to the straight lines $x \sec\alpha + y \operatorname{cosec}\alpha = a$ and $x \cos\alpha - y \sin\alpha = a \cos 2\alpha$, then prove that $4p^2 + q^2 = a^2$.

31. Show that the lines represented by $(lx + my)^2 - 3(mx - ly)^2 = 0$ and $lx + my + n = 0$ form an equilateral triangle with area $\frac{n^2}{\sqrt{3}(l^2 + m^2)}$.

32. Find the condition for the chord $lx + my = 1$ of the curve $x^2 + y^2 = a^2$ to subtend a right angle at origin.

33. If a ray makes an angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, then find $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$.

34. If $y = \operatorname{Tan}^{-1}\left(\frac{2x}{1 - x^2}\right) + \operatorname{Tan}^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) - \operatorname{Tan}^{-1}\left(\frac{4x - 4x^3}{1 - 6x^2 + 6x^4}\right)$, then prove that $\frac{dy}{dx} = \frac{1}{1 + x^2}$.

35. Find the derivative of the function $f(x) = \sin^{-1}\left(\frac{b + a\sin x}{a + b\sin x}\right)$.

36. Find the angle between the curves $x + y + 2 = 0$ and $x^2 + y^2 - 10y = 0$.

37. A wire of length l is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of the pieces of the wire respectively so that the sum of the areas is the least?

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