1. REAL NUMBERS

Rational number:

The number, which is written in the form of $\frac{p}{q}$ where, p, q are integers and $q \neq 0$ is called rational number. It is denoted by Q.

Irrational number:

The number, which is not rational, is called irrational number. It is denoted by Q' or S.

Prime number:

The number which has only two factors 1 and itself is called prime number.

2, 3, 5, 7 .... Etc.

Composite number:

The number which has more than two factors is called composite number.

4, 6, 8, 9, 10 ... etc.

Co-prime numbers:

Two numbers are said to be co-prime numbers, if they have no common factor except 1.

Ex: (1, 2), (3, 4), (4, 7) ...etc.

Note:

(i) HCF of co – prime numbers(prime numbers) is always 1

(ii) LCM of co – prime numbers(prime numbers) is their product

Fundamental theorem of arithmetic:

Every composite number can be expressed as product of primes, and this factorisation is unique, apart from the order in which prime factors occurs.

Ex: $12 = 2 \times 2 \times 3 = 2^2 \times 3$, $15 = 3 \times 5$ and so on.

To find LCM and HCF by using prime factorisation method:

H.C.F = product of the smallest power of each common prime factors of given numbers.

L.C.M = product of the greatest power of each prime factor of given numbers.

Ex: Find LCM and HCF of 12 and 18 by using prime factoriasation Method

Sol: Prime factors of $12 = 2 \times 2 \times 3 = 2^2 \times 3$

Prime factors of $18 = 2 \times 3 \times 3 = 2 \times 3^2$

$\text{HCF} = 2 \times 3 = 6$

$\text{LCM} = 2^2 \times 3^2 = 4 \times 9$

$= 36$

Relationship between L.C.M and H.C.F of two numbers:

For any two positive integers 'a' and 'b'

H.C.F (a, b) × L.C.M (a, b) = a × b

Ex: Let the numbers be 4 and 12

H.C.F (4, 12) = 4 and L.C.M (4, 12) = 12

H.C.F (4, 12) × L.C.M (4, 12) = 48

$a \times b = 4 \times 12 = 48$

$$\text{HCF}(a, b) = \frac{a \times b}{\text{LCM}(a,b)}$$

$$\text{LCM}(a, b) = \frac{a \times b}{\text{HCF}(a,b)}$$

$$b = \frac{\text{HCF}(a, b) \times \text{LCM}(a, b)}{a}$$

Exercise 1.1

1. Express each number as a product of its prime factors

(i) 140 (ii) 156 (iii) 3825

(iv) 5005 (v) 7429

Solution:

(i) 140

$140 = 2 \times 2 \times 5 \times 7$

$= 2^2 \times 5 \times 7$

2	140
2	70
5	35
7	7
	1

(ii) 156

$156 = 2 \times 2 \times 3 \times 13$

$= 2^2 \times 3 \times 13$

2	156
2	78
3	39
13	13
	1

(iii) 3825

$3825 = 3 \times 3 \times 5 \times 5 \times 17$

$= 3^2 \times 5^2 \times 17$

3	3825
3	1275
5	425
5	85
	17

(iv) 5005

$5005 = 5 \times 7 \times 11 \times 13$

5	5005
7	1001
11	143
13	13
	1

(v) 7429

$7429 = 17 \times 19 \times 23$

17	7429
19	437
	23

2. Find the LCM and HCF of the following pairs of integers and verify that

$LCM \times HCF =$ product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Solution:

(i) 26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

$HCF\ (26, 91) = 13$

$LCM\ (26, 91) = 2 \times 7 \times 13 = 182$

Verification:

$26 \times 91 = 2366$

$HCF\ (26, 91) \times LCM\ (26, 91) = 13 \times 182 = 2366$

$\therefore$ Product of 26 and 91 = product of HCF (26, 91) and LCM (26, 91)

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$HCF\ (510, 92) = 2$

$LCM\ (510, 92) = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Verification:

$510 \times 92 = 46920$

$HCF\ (510, 92) \times LCM\ (510, 92) = 2 \times 23460 = 46920$

$\therefore$ Product of 510 and 92 = product of HCF (510, 92) and LCM (510, 92)

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$HCF\ (336, 54) = 2 \times 3 = 6$

$\mathbf{LCM\ (336, 54) = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024}$

Verification:

$336 \times 54 = 18{,}144$

$HCF\ (336, 54) \times LCM\ (336, 54) = 6 \times 3024 = 18{,}144$

$\therefore$ Product of 26 and 91 = product of HCF (336, 54) and LCM (336, 54)

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Solution:

(i) 12, 15 and 21

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

HCF $(12, 15, 21) = 3$

LCM $(12, 15, 21) = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29

Since 17, 23 and 29 are prime numbers

HCF $(17, 23, 29) = 1$

LCM $(17, 23, 29) = 17 \times 23 \times 29 = 11339$

(iii) 8, 9 and 25

$8 = 2 \times 2 \times 2 = 2^3$

$9 = 3 \times 3 = 3^2$

$25 = 5 \times 5 = 5^2$

8, 9 and 25 are co – prime numbers

HCF $(8, 9, 25) = 1$

LCM $(8, 9, 25) = 8 \times 9 \times 25 = 1800$

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Solution:

We know that, product of two numbers = product of their LCM and HCF

Given HCF $(306, 657) = 9$

$$\text{LCM }(306, 657) = \frac{306 \times 657}{\text{HCF }(306,657)}$$

$$= \frac{306 \times 657}{9}$$

$$= 34 \times 657$$

$$= 22338$$

$\therefore$ LCM $(306, 657) = 22338$

5. Check whether 6^n can end with the digit 0 for any natural number n.

Solution:

If the number 6^n, for any n, were to end with the digit zero, then it would be divisible by 5.

That is, the prime factorisation of 6^n would contain the prime 5.

This is not possible because $6^n = (2 \times 3)^n$; so the primes in the factorisation of 6^n are 2, 3.

Since, 5 is not present in the prime factorisation of 6^n

$\therefore$ there is no natural number n for which 6^n ends with the digit zero.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution:

$$\begin{array}{r|l} 2 & 78 \\ \hline 3 & 39 \\ \hline & 13 \end{array}$$

$7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1)$

$=13(77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 3 \times 2$

Product of prime numbers

By the fundamental theorem of Arithmetic

$7 \times 11 \times 13 + 13$ is a composite number

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5\ (7 \times 6 \times\ 4 \times 3 \times 2 \times 1 + 1)$

$=5(1008 + 1)$

$= 5 \times 1009$

Product of prime numbers

By the fundamental theorem of Arithmetic

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is a composite number

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution:

Given, Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same

We have to find out, after how many minutes they will meet again at the starting point

In this situation we have to find LCM (12, 18)

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

LCM $(12, 18) = 2^2 \times 3^2 = 36$

$\therefore$ Sonia and Ravi will meet after 36 minute

Irrational Number:

A number cannot be written in the form of $\frac{p}{q}$ where p, q are integers $q \neq 0$ is an irrational number. It is denoted by Q' or S.

Ex: $\sqrt{3}, \sqrt{5}, \sqrt{7}$ .. etc.

$\sqrt{p}$ is always an irrational when, p is not a perfect square.

'p' is a prime number and 'a' is a positive integer, if p divides a^2, then p divides a

Proof by contradiction Method:

Proof by contradiction is a method used in mathematics to prove the truth of a statement by assuming the opposite (negation) of what is to be proven and then showing that this assumption leads to a contradiction, thereby establishing the original statement as true.

Here's how it works in a nutshell

Assume the opposite: Start by assuming that the statement you want to prove (let's call it X) is false. This means you assume '~X', where "~" represents "not".

Derive a contradiction: Then, using logical reasoning and the assumptions made in step 1, you deduce something that is clearly impossible or contradictory.

Conclude: Since assuming ~X leads to a contradiction, the original statement X must be true.

This method relies on the principle of excluded middle, which states that a statement must be either true or false, with no middle ground. By showing that assuming the opposite leads to a contradiction, you demonstrate that the original statement cannot be false, so it must be true.

EX: Prove that $\sqrt{2}$ is an irrational number

Solution:

Let assume that $\sqrt{2}$ a rational number

For any two integers r and s, $\sqrt{2}$ can be written as

$\sqrt{2} = \frac{r}{s}$

If r, s have a common factor, then divide both r and s by their HCF to get

$\sqrt{2} = \frac{a}{b}$, where a, b are co-primes

$\Rightarrow a = \sqrt{2}\, b$

Squaring on both sides

$a^2 = 2b^2$ ------------------- (1)

$\frac{a^2}{2} = b^2$

2 divides a^2

Since 2 is prime, 2 divides a

2 is the factor of a

Let $a = 2c$, where c is any ineger

$a^2 = 4\ c^2$

$2b^2 = 4\ c^2$ [From (1)]

$b^2 = 2c^2$ ------------------- (1)

$\frac{b^2}{2} = c^2$

2 divides b^2

Since 2 is prime, 2 divides b

2 is the factor of b

⇒ 2 is the common factor of both a and b

This is a contradiction to the fact that a, b are co – primes

Our assumption is wrong

∴ $\sqrt{2}$ is an irrational number

Exercise 1.2

1. Prove that $\sqrt{5}$ is an irrational number

Solution:

Let assume that $\sqrt{5}$ a rational number

For any two integers r and s, $\sqrt{5}$ can be written as

$\sqrt{5} = \frac{r}{s}$

If r, s have a common factor, then divide both r and s by their HCF to get

$\sqrt{5} = \frac{a}{b}$, where a, b are co-primes

⇒ $a = \sqrt{5}\ b$

Squaring on both sides

$a^2 = 5b^2$ ------------------- (1)

$\frac{a^2}{5} = b^2$

5 divides a^2

Since 5 is prime, 5 divides a

5 is the factor of a

Let $a = 5c$, where c is any ineger

$a^2 = 25\ c^2$

$5b^2 = 25\ c^2$ [From (1)]

$b^2 = 5c^2$ ------------------- (1)

$\frac{b^2}{5} = c^2$

5 divides b^2

Since 5 is prime, 5 divides b

5 is the factor of b
$\Rightarrow$ 5 is the common factor of both a and b
This is a contradiction to the fact that a, b are co – primes
Our assumption is wrong
$\therefore\ \sqrt{5}$ is an irrational number

2. Prove that $3 + 2\sqrt{5}$ is irrational

Solution:

Let assume that $3 + 2\sqrt{5}$ a rational number
For any two integers a and b ($\neq 0$), $3 + 2\sqrt{5}$ can be written as

$$3 + 2\sqrt{5} = \frac{a}{b}$$
$$2\sqrt{5} = \frac{a}{b} - 3$$
$$2\sqrt{5} = \frac{a - 3b}{b}$$
$$\sqrt{5} = \frac{a - 3b}{2b}$$

Since, 2, 3, a, b are integers $\frac{a-3b}{2b}$ is a rational number and so, $\sqrt{5}$ is also rational number
This is a contradiction to the fact that $\sqrt{5}$ is an irrational number
Our assumption is wrong
$\therefore 3 + 2\sqrt{5}$ is an irrational number

3. Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Solution:

(i) Let assume that $\frac{1}{\sqrt{2}}$ a rational number
For any two integers a and b ($\neq 0$), $3 + 2\sqrt{5}$ can be written as

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$
$$a\sqrt{2} = b \text{ (by cross multiplication)}$$
$$\sqrt{2} = \frac{b}{a}$$

Since, a, b are integers $\frac{b}{a}$ is a rational number and so, $\sqrt{2}$ is also rational number
This is a contradiction to the fact that $\sqrt{2}$ is an irrational number
Our assumption is wrong
$\therefore \frac{1}{\sqrt{2}}$ is an irrational number

(ii) Let assume that $7\sqrt{5}$ a rational number
For any two integers a and b ($\neq 0$), $7\sqrt{5}$ can be written as

$$7\sqrt{5} = \frac{a}{b}$$

$\sqrt{5} = \frac{a}{7b}$

Since,7, a, b are integers $\frac{a}{7b}$ is a rational number and so, $\sqrt{5}$ is also rational number

This is a contradiction to the fact that $\sqrt{5}$ is an irrational number

Our assumption is wrong

$\therefore 7\sqrt{5}$ is an irrational number

(iii) Let assume that $6 + \sqrt{2}$ a rational number

For any two integers a and b $(\neq 0)$, $6 + \sqrt{2}$ can be written as

$6 + \sqrt{2} = \frac{a}{b}$

$\sqrt{2} = \frac{a}{b} - 6$

$\sqrt{2} = \frac{a - 6b}{b}$

Since, 6, a, b are integers $\frac{a - 6b}{b}$ is a rational number and so, $\sqrt{2}$ is also rational number

This is a contradiction to the fact that $\sqrt{2}$ is an irrational number

Our assumption is wrong

$\therefore 6 + \sqrt{2}$ is an irrational number