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TS

X CLASS

Mathematics

SOLVED QUESTION PAPER-3

By Satyam

SUBJECT: MATHEMATICS
CLASS: X

SOLVED QUESTION PAPER - 3

TIME: 3 Hours
TOTAL MARKS: 80

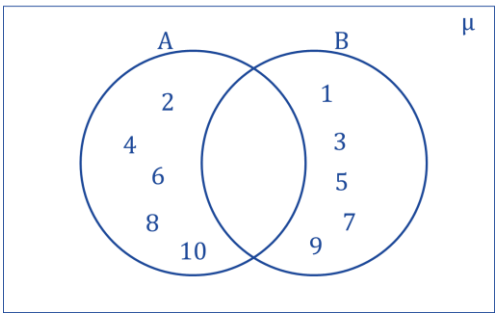
PART – A

SECTION – I (6 × 2 = 12)

1. If A= {2, 4, 6, 8, 10} and B= {1, 3, 5, 7, 9}, then represent these sets in a Venn diagram.

Answer:

Given A= {2, 4, 6, 8,1 0} and B= {1, 3, 5, 7, 9},



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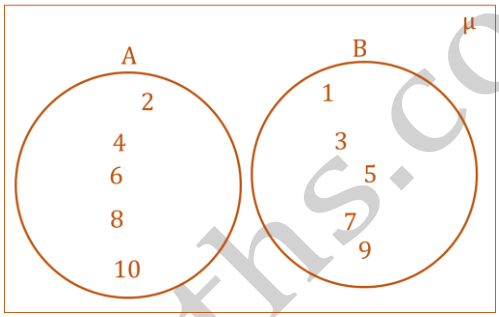


2. Find the zeroes of the quadratic polynomial $P(x) = 4x^2 - 4x + 1$.

Answer:

Given $P(x) = 4x^2 - 4x + 1$

For the zeroes of the polynomial $P(x) = 0$

$4x^2 - 4x + 1 = 0$

$4x^2 - 2x - 2x + 1 = 0$

$2x(2x - 1) - 1(2x - 1) = 0$

$(2x - 1)(2x - 1) = 0$

$x = \frac{1}{2}, \frac{1}{2}$

∴The zeroes of the polynomial are $\frac{1}{2}, \frac{1}{2}$

3. Find the centroid of the triangle whose vertices are A (0, 0), B (1, 4) and C (2, -2).

Answer:

Centroid of the triangle $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$= \left(\frac{0+1+2}{3}, \frac{0+4-2}{3}\right)$

$= \left(\frac{3}{3}, \frac{2}{3}\right)$

$= \left(1, \frac{2}{3}\right)$

∴ The centroid of the triangle is $\left(1, \frac{2}{3}\right)$

4. ΔABC~ ΔDEF, if DE=2AB and BC=3cm, then EF is equal to 3cm. Are you agree this? Justify your answer with appropriate reasons.

Answer:

I am not agreeing with this, because

Given: ΔABC ~ Δ DEF in which DE = 2 AB and BC = 3 cm.

By Rule of triangle SSS (side-side-side) we have:

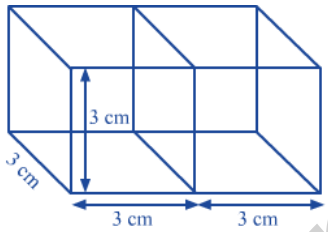
$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{AB}{2AB} = \frac{3}{EF}$

EF = 6cm

5. Two cubes each of volume $27cm^3$ are joined end to end together. Find the total surface area of the resulting cuboid.

Answer:



Volume of one cube, $v=27\ cm^3$

side of a cube $s=\sqrt[3]{27}=3$ cm

Two cubes of the same volume joined together to form a cuboid.

$\therefore$ the length of cuboid, $L=2s=6$ cm

Width of cuboid (B) = Height of cuboid (H) =3 cm

Surface Area of cuboid $=2(LB+BH+LH) = 2(6\times3+3\times3+6\times3) = 2(18+9+18) = 90\ cm^2$

6. Write the formula to find the mode of the grouped data and explain each term in it.

Answer:

The formula to find the mode of the grouped data is: $\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

l = lower class limit of modal class

f_0 =frequency of class proceeding to modal class

f_1 = frequency of modal class

f_2 = frequency of class succeeding to modal class

h = class size

SECTION – II (6 × 4 = 24)

7. If A= {x: x is a factor of 12} and B= {x: x is a factor of 30}, then verify the relation between n(A), n(B), n(A∩B) and n (A∪B).

Answer:

Given A= {x: x is a factor of 12}

$A = \{1, 2, 3, 4, 6, 12\}$

B= {x: x is a factor of 30}

$B = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$n(A) = 6, n(B) = 8$

$A \cap B = \{1, 2, 3, 6\}$

$\Rightarrow\ n(A \cap B) = 4$

$A\cup B = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 30\}$

$n(A\cup B) = 10$

$n(A) + n(B) - n(A \cap B) = 6 + 8 - 4$

$= 10$

$\therefore n(A\cup B) = n(A) + n(B) - n(A \cap B)$

8. Seven times a two-digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3. Find the number.

Answer:

Let numbers be x at ones place & y at tens place so,

$10y+x$ is the original number.

Reversed number is $=10x + y$

According to the question,

$7(10y + x) = 4(10x + y)$

$\Rightarrow x = 2y$ …….. (i)

Now,
Given, $x - y = 3$
From equation (i) $2y = x$ substitute in above equation.
$2y - y = 3$
$x = 3$
$y = 2y = 2(3) = 6$
$\therefore$ Required original number = $10y + x = 10(3) + 6 = 36$.

9. If in an A.P. 7 times of 7th term is equal to 11 times of 11th term. Then show that 18th term is equal to zero.

Answer:

7 times the 7th term of an A.P. is equal to 11 times its 11th term
$\Rightarrow 7 \times a_7 = 11 \times a_{11}$
$\Rightarrow 7[a + (7 - 1)d] = 11[a + (11 - 1)d]$
$\Rightarrow 7[a + 6d] = 11[a + 10d]$
$\Rightarrow 7a + 42d = 11a + 110d$
$\Rightarrow 11a - 7a + 110d - 42d = 0$
$\Rightarrow 4a + 68d = 0$
$\Rightarrow 4(a + 17d) = 0$
$\Rightarrow a + 17d = 0$
$\therefore a + 17d = 0$..... (i)
$\Rightarrow a_{18} = a + (18-1)d$
$\Rightarrow a_{18} = a + 17d$
$\therefore a_{18} = 0$ [from (i)]

10. ΔABC and ΔPQR are similar triangles. If AB = 4 cm, BC = 5 cm, AC = 2 cm, and PQ = 8 cm, find the perimeter of the ΔPQR.

Answer:

ABC and ΔPQR are similar triangles. AB = 4 cm, BC = 5 cm, AC = 2 cm, and PQ = 8 cm
We know that corresponding sides of similar triangles are proportional.
$\therefore \frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC}$
$\therefore \frac{8}{4} = \frac{QR}{5} = \frac{PR}{2}$
QR = 10 cm and PR = 4 cm
Perimeter of ΔPQR = PQ + QR + PR
Perimeter of $\Delta PQR = 8 + 10 + 4 = 22$ cm

11. If $\sqrt{3}\tan\theta = 3\sin\theta$, then find the value of $\sin^2\theta - \cos^2\theta$.

Answer:

Given $\sqrt{3}\tan\theta = 3\sin\theta$
$\sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$
$\frac{\sqrt{3}}{\cos\theta} = 3$
$\cos\theta = \frac{1}{\sqrt{3}}$
$\sin^2\theta = 1 - \cos^2\theta$
$= 1 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= 1 - \frac{1}{3} = \frac{2}{3}$

$\sin\theta = \sqrt{\frac{2}{3}}$

$\sin^2\theta - \cos^2\theta = \left(\sqrt{\frac{2}{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

12. There are mobile numbers of 20 teachers, 16 lawyers and 14 doctors on a page. If a number is selected at random, then what is the probability of selecting a mobile number of (i) a teacher (ii) a person who is not a teacher.

Answer:

Number of teachers = 20

Number of lawyers = 16

Number of doctors = 14

Total persons = 50

(i) Probability of selecting a mobile number of a teacher $= \frac{20}{50} = \frac{2}{5}$

(ii) Probability of selecting a mobile number of a person who is not a teacher $= \frac{30}{50} = \frac{3}{5}$

SECTION – III (4 × 6 = 24)

13. $x^2 + y^2 = 6xy$, then prove that $2\log(x+y) = \log x + \log y + 3\log 2$.

Answer:

Given $x^2 + y^2 = 6xy$

Add 2xy on both sides

$x^2 + y^2 + 2xy = 6xy + 2xy$

$(x+y)^2 = 8xy$

Apply log on both sides

$\log(x+y)^2 = \log 8xy$

$2\log(x+y) = \log 8 + \log x + \log y$

$2\log(x+y) = \log 2^3 + \log x + \log y$

$2\log(x+y) = 3\log 2 + \log x + \log y$

Hence proved

14. Find the zeroes of the quadratic polynomial $P(x) = x^2 + 7x + 12$ and verify the relation between zeroes and its coefficients.

Answer:

Given, $P(x) = x^2 + 7x + 12$

For zeros of the polynomial $P(x) = 0$

$x^2 + 7x + 12 = 0$

$x^2 + 3x + 4x + 12 = 0$

$x(x+3) + 4(x+3) = 0$

$(x+3)(x+4) = 0$

$x = -3$ or $x = -4$

verification,

let $\alpha = -3, \beta = -4$

Sum of roots $= \alpha + \beta$

$= -3 - 4$

$= -7$

$= \frac{-7}{1}$

$= \frac{-(\text{x coefficient})}{x^2 \text{ coefficient}}$

Product of roots $= \alpha \times \beta$

$= -3 \times -4$

$= 12$

$= \frac{12}{1}$

$= \frac{\text{constant}}{x^2 \text{ coefficient}}$

Hence proved

15. The hypotenuse of a right-angled triangle is 6m more than the twice the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.

Answer:

Let the shortest side of the triangle be x m.

Then, hypotenuse$=(2x + 6)$ m

Third side$=(2x + 6)-2=(2x + 4)$ m

By Pythagoras' Theorem,

$(2x+6)^2 = (2x+4)^2 + x^2$

$\Rightarrow 4x^2 + 24x + 36 = 4x^2 + 16x + 16 + x^2$

$\Rightarrow x^2 - 8x - 20 = 0$

$\Rightarrow x^2 - 10x + 2x - 20 = 0$

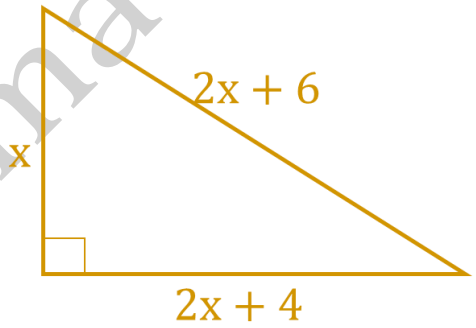
$\Rightarrow (x-10)(x+2) = 0$

$\Rightarrow x = 10$ or -2

Since x cannot be negative

$x = 10$

$\therefore$ the sides are 10, 24, 26



16. From 'O' point 'P', which is at a distance of 13cm from the centre of circle of radius 5cm, the pair of tangents PQ and PR are drawn to the circle, then find the area of the quadrilateral PQOR

Answer:

Draw a circle of radius 5 cm having centre O. P is a point at a distance of 13 cm from O. A pair of tangents PQ and PR are drawn

[Since, QP is a tangent line]

$\therefore OQ \perp QP$

In right angled

ΔPQO, $OP^2 = OQ^2 + QP^2$

$\Rightarrow 13^2 = 5^2 + QP^2$

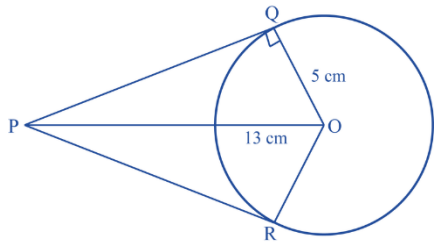
$\Rightarrow QP^2 = 169 - 25 = 144$

$QP = 12$cm

Now,

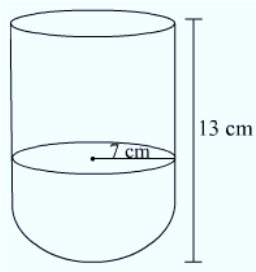
Area of $\Delta OQP = 12 \times QP \times QO = 12 \times 12 \times 5 = 30$ cm^2

$\therefore$ Area of quadrilateral $QORP = 2 \times$ Area of $\Delta OQP = 2 \times 30 = 60$ cm^2



17. A vessel is in the shape of hemisphere and a cylinder surmounted on it. If the diameter of the vessel is 14cm and complete height of vessel is 13cm, then find volume of vessel.

Answer:



Let the radius and height of cylinder is r cm and h cm respectively.
Diameter of the hemisphere bowl =14 cm
Radius of the hemispherical bowl = Radius of the cylinder
=r=14/2 cm=7 cm
Total height of the vessel=13cm
Height of the cylinder= Total height of the vessel− Radius of the hemispherical bowl
=13 cm−7cm=6cm
Volume of the vessel = volume of cylinder + volume of hemi sphere

$$= \pi r_1^2 h_1 + \frac{2}{3}\pi r^3$$

$$= \frac{22}{7} \times 7 \times 7 \times 6 + \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= 924 + \frac{2156}{3}$$

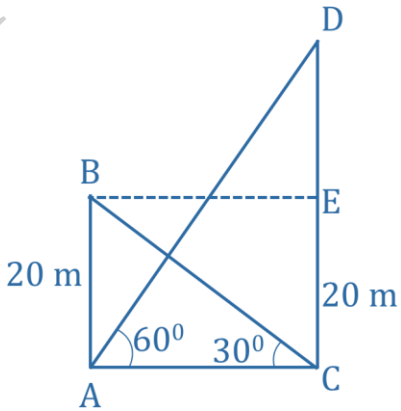
$$= \frac{2772+2156}{3}$$

$$= \frac{4928}{3} cm^3$$

18. A 20m high vertical pole and a vertical tower are on the same level ground in such a way that the angle of elevation of the top of the tower, as seen from the foot of the pole is 60^0 and the angle of elevation of the top of the pole as seen from the foot of the tower is 30^0. Find: (i) the height of the tower; (ii) the horizontal distance between the pole and the tower.

Answer:

Let AB is the pole and CD is the tower
AB = 20m
AB = CE = 20m
$\angle DAC = 60^0$ and $\angle ACB = 30^0$
From Δ ABC



$\text{Tan } 30^0 = \frac{20}{AC}$

$\frac{1}{\sqrt{3}} = \frac{20}{AC}$

$AC = 20\sqrt{3}$

From Δ ACD

$\text{Tan } 60^0 = \frac{DC}{AC}$

$\sqrt{3} = \frac{20 + DE}{AC}$

$\sqrt{3}AC = 20 + DE$

$\sqrt{3}(20\sqrt{3}) = 20 + DE$

$60 = 20 + DE$

$DE = 60 - 20 = 40$

Height of the tower = 40m + 20m = 60 m

Horizontal distance between the pole and the tower $=20\sqrt{3}$m

PART – B

Chose the correct answer **20 × 1=20**

1. HCF of 26 and 91 is (c)
   a) 15 b) 19 c) 13 d) 11
2. Which of the following is not a set builder form of A={2,4,6,8,10}? (a)
   a) A= {x: x is an even number} b) A= {x: x=2n, n≤5,n∈N}
   c) A= {x: x is the list of first five even numbers} d) A= {x: x is an even number, x< 11}
3. Which of the following are not infinite sets? A= {x: x ∈ N}: B={x: x is a multiple of 5}, C= {x: x is a factor of 15} (c)
   a) Only A b) Only B c) Only C d) Only B and C
4. If R= {2,3,5,7}, then (c)
   a) 2∉ R b) 4 ∈ R c) 5 ∈ R d) 7 ∉ R
5. If $P(x) = 3x^4 - 5x^3 + x^2 + 8$, then the value of P(-1) is (b)
   a) 2 b) 17 c) 15 d) 18
6. If $x = \log_2 8$ and $y = \log_7 49$ is the solution of $x + y = a$, then value of 'a' is (d)
   a) 3 b)2 c) 4 d) 5
7. If the roots of $x^2 - bx + c=0$ are two consecutive integers, then (b)
   a) $b^2 - 4c = 0$ b) $b^2 - 4c = 1$ c) $b^2 + 4c = 1$ d) $b^2 + 4c = 0$
8. Which term of the A.P. 100, 90, 80, ... is zero? (c)
   a) 9th b) 10th c) 11th d) 12th
9. Slope of the line joining the points (5, 3) and (6, 2) is (a)
   a) –1 b) –2 c) –3 d) –4
10. The ratio in which the centroid divides the median from the vertex of the triangle is (d)
    a) 1:2 b) 1:1 c) 1:3 d) 2:1
11. If 'a' is a side of an equilateral triangle then what is its altitude? (c)
    a) $\frac{\sqrt{3}}{2}a^2$ b) $\frac{\sqrt{3}}{2}a^3$ c) $\frac{\sqrt{3}}{2}a$ d) $\frac{1}{\sqrt{3}}a$
12. The length of tangent from 'C' point 15 cm away from the centre of a circle of radius 9 cm is (b)
    a) 10 cm b) 12cm c) 14 cm d) 13cm
13. The surface areas of two spheres are in the ratio 4:9 then, ratio of their volumes (b)
    a) 27 : 8 b) 8 : 27 c) 1 : 4 d) 2 : 3
14. The value of $(\sin 30^0 + \cos 30^0) - (\sin 60^0 + \cos 60^0)$ is (b)
    a) 2 b) 0 c) 1 d) 3
15. If Tan θ is expressed in Sin θ as (a)
    a) $\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ b) $1 - \sin^2\theta$ c) $\frac{1}{\sqrt{1+\sin^2\theta}}$ d) $1 + \sin^2\theta$
16. If a person observes the top of a tree at an angle of elevation 45^0 from 20cm distance from the fort of the tree, then height of the tree is (d)
    a) 40 cm b) 10 cm c) 30 cm d) 20 cm
17. What is the probability of a sure event? (b)
    a) 0 b) 1 c) 2 d) – 1
18. What is the probability getting a prime number when a dice is rolled? (a)
    a) 0.5 b) 2.6 c) 0.6 d) 0.3
19. Mid values are used to find which of the following central tendencies (b)
    a) range b) mean c) mode d) median
20. Mode of first 10 natural numbers has (d)
    a) One mode b) Two Modes c) Three Modes d) No Mode