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CBSE

X CLASS

Mathematics

SOLVED QUESTION PAPERS

By Satyam

Section A

※This section consists of 20 MCQ'S questions of 1 mark each

1. If α and β are the zeroes of the polynomial $x^2 + 2x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

a) – 2 b) 2 c) 0 d) 1

Option (a) is Correct

Solution: Given polynomial is $x^2 + 2x + 1$

$\alpha + \beta = -2, \alpha\beta = 1$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-2}{1} = -2$

2. The roots of the quadratic equation $x^2 - 0.04 = 0$ are

a) ± 0.2 b) ± 0.02 c) 0.4 d) 2

Option (a) is Correct

Solution: Given equation is $x^2 - 0.04 = 0$

$x = \sqrt{0.04} = \pm 0.2$

3. In the given figure, PA is a tangent from an external point P to a circle with centre O. If $\angle POB = 115^0$, then the value $\angle APO$ is

a) 25^0 b) 20^0 c) 30^0 d) 65^0

Option (a) is Correct

Solution: $\angle POB = 115^0$

$\angle POA = 180^0 - 115^0 = 65^0$

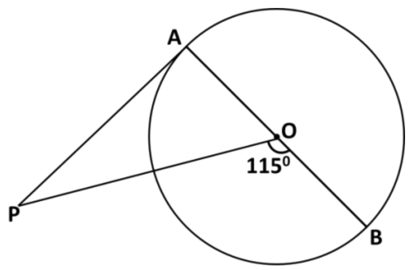
In ΔPOA

$\angle APO + \angle POA + \angle OAP = 180^0$

$\angle APO + 65^0 + 90^0 = 180^0$

$\angle APO + 155^0 = 180^0$

$\angle APO = 180^0 - 155^0 = 25^0$



4. In an AP if $d = -4$, $n = 7$ and $a_n = 4$, then a is equal to

a) 6 b) 7 c) 20 d) 28

Option (d) is Correct

Solution: Given $d = -4$, $n = 7$ and $a_n = 4$,

$a_n = 4 \Rightarrow a + (n-1)d = 4$

$\Rightarrow a + 6d = 4 \Rightarrow a + 6(-4) = 4$

$\Rightarrow a - 24 = 4$

$\Rightarrow a = 4 + 24 = 28$

5. A bag contains 3 red and 2 blue marbles. if a marble is drawn at random, then the probability of drawing a blue marble is

a) $\frac{2}{5}$ b) $\frac{1}{4}$ c) $\frac{3}{5}$ d) $\frac{2}{3}$

Option (a) is Correct

Solution: No. of red marbles = 5; No. of blue marbles = 2

Total possible outcomes = 5

Probability of drawing a blue marble is = $\frac{2}{5}$ 3

6. 225 is can be expressed as

a) 5×3^2 b) $5^2 \times 3$ c) $5^2 \times 3^2$ d) $5^3 \times 3$

3	225
3	75
5	25
5	5

Option (c) is Correct

Solution: $225 = 3 \times 3 \times 5 \times 5$

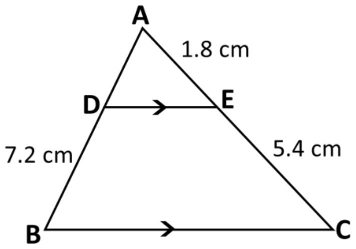
$= 5^2 \times 3^2$

7. In given figure DE ∥ BC. Find the length of side AD, given that AE = 1.8 cm, BD = 7.2 cm and CE = 5.4 cm.

a) 2.4 cm b) 2.2 cm c) 3.2 cm d) 3.4 cm

Option (a) is Correct

Solution: In Δ ABC, DE ∥ BC



By basic proportionality theorem

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{AD}{7.2} = \frac{1.8}{5.4} = \frac{18}{54} = \frac{1}{3}$$

$$\frac{AD}{7.2} = \frac{1}{3}$$

AD = 2.4 cm

8. Consider the following distribution:

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

The frequency of class 30 – 40 is:

a) 3 b) 4 c) 48 d) 51

Option (a) is Correct

Solution:

Marks obtained	Number of students	Frequency
0 – 10	63	5
10 – 20	58	3
20 – 30	55	4
30 – 40	51	3
40 – 50	48	6
50 – 60	42	42

9. If $\cos 9\alpha = \sin \alpha$ and $9\alpha < 90^0$, then the value of $\tan 5\alpha$ is

a) $\frac{1}{\sqrt{3}}$ b) $\sqrt{3}$ c) 1 d) 0

Option (c) is Correct

Solution: Given $\cos 9\alpha = \sin \alpha$ and $9\alpha < 90^0$

$\cos 9\alpha = \cos (90 - \alpha)$

$9\alpha = 90 - \alpha$

$10\alpha = 90$

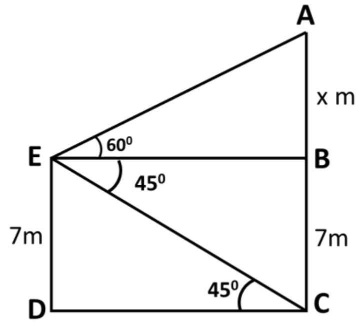
$\alpha = 9$

$\tan 5\alpha = \tan (5 \times 9) = \tan 45 = 1$

10. From the top of a 7m high building the angle of elevation of the top of a cable tower is 60^0 and the angle of depression of its foot is 45^0, then the height of the tower is

a) 14.124 m b) 17.124 m c) 19.124 m d) 1.124 m

Option (c) is Correct



Solution: In Δ EDC, $\tan 45^0 = \frac{ED}{DC}$

$1 = \frac{7}{DC}$

$DC = 7m$

$\Longrightarrow DC = EB = 7m$

In Δ ABE, $\tan 60^0 = \frac{AB}{BE}$

$\sqrt{3} = \frac{x}{7}$

$x = 7\sqrt{3}$ m

$\Longrightarrow DC = EB = 7m$

the height of the tower $= AB + BC = 7m + 7\sqrt{3}$ m

$= 7\,(1+\sqrt{3}\,)m$

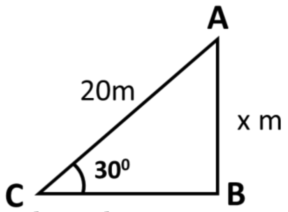
$= 7\,(2.732)m$

$= 19.124$ m

11. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. If the angle made by the rope with the ground level is 30^0, then what is the height of the pole?

a) 20m b) 8m c) 10m d) 6m

Option (c) is Correct



Solution: In Δ ABC, $\sin 30^0 = \frac{AB}{AC}$

$\frac{1}{2} = \frac{x}{20}$

$x = 10m$

12. The maximum number of zeroes a cubic polynomial can have is

a) 1 b) 4 c) 2 d) 3

Option (d) is Correct

Solution: In a cubic polynomial number of zeros = 3

Why because degree of the polynomial = 3.

13. If triangle ABC is similar to triangle DEF such that 2 AB = DE and BC = 8 cm, then find EF.

a) 16 cm b) 14 cm c) 12 cm d) 15 cm

Option (a) is Correct

Solution: Given $\Delta ABC \sim \Delta\, DEF$ 2 AB = DE and BC = 8 cm

$2\,AB = DE \Longrightarrow \frac{AB}{DE} = \frac{1}{2}$

Since, $\Delta ABC \sim \Delta\, DEF$

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\frac{1}{2} = \frac{8}{EF} \Longrightarrow EF = 16$

14. A sphere is melted and half of the molten liquid is used to form 11 identical cubes, whereas the remaining half is used to form 7 identical smaller spheres. The ratio of the side of the cube to the radius of the new small sphere is

a) $\left(\frac{4}{3}\right)^{1/3}$ b) $\left(\frac{8}{3}\right)^{1/3}$ c) $(3)^{1/3}$ d) 2

Option (b) is Correct

Solution:

Volume of 11 small cubes = Volume of 7 small spheres

$$11\, a^3 = 7\left(\frac{4}{3}\pi\, r^3\right)$$

$$11\, a^3 = 7 \times \frac{4}{3} \times \frac{22}{7}\ r^3$$

$$a^3 = \frac{8}{3}\ r^3$$

$$\frac{a^3}{r^3} = \frac{8}{3} \Rightarrow \frac{a}{r} = \left(\frac{8}{3}\right)^{1/3}$$

15. Ratio of volumes of two cones with same radii is

a) $h_1 : h_2$ b) $r_1 : r_2$ c) $s_1 : s_2$ d) $l_1 : l_2$

Option (a) is Correct

Solution:

$$\frac{1}{3}\pi\, r^2\, h_1 : \frac{1}{3}\pi\, r^2\, h_2 \;= h_1 : h_2$$

16. In the formula $a + h\left(\frac{\sum f_i \mu_i}{\sum f_i}\right)$, for finding the mean of un grouped frequency distribution, μ_i is equal to

a) $\frac{x_i + a}{h}$ b) $h\,(x_i - a)$ c) $\frac{x_i - a}{h}$ d) $\frac{a - x_i}{h}$

Option (c) is Correct

17. If the probability of an event is p, then the probability of its complementary event will be

a) $p - 1$ b) p c) $1 - p$ d) $1 - \frac{1}{p}$

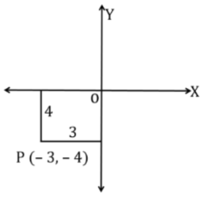
Option (c) is Correct

18. The distance of the point P (– 3, – 4) from the X – axis is

a) 3 b) – 3 c) 4 d) – 4

Option (c) is Correct

Solution:



19. Assertion (A): Pair of linear equations $9x + 3y + 12 = 0$, $8x + 6y + 24 = 0$ have infinitely many solutions.

Reason (R): Pair of linear equations $a_1 x + b_1 y + c_1 = 0$, $a_2 x + b_2 y + c_2 = 0$ have infinitely many solutions. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

a) both A and R are true and R is the correct explanation of A
b) both A and R are true and R is the not correct explanation of A
c) A is true, R is false
d) A is false, R is true

Option (d) is Correct

Solution:

Given equations are $9x + 3y + 12 = 0, 8x + 6y + 24 = 0$

$$\frac{a_1}{a_2} = \frac{9}{8}; \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Pair of linear equations $9x + 3y + 12 = 0, 8x + 6y + 24 = 0$ have finite solutions

20. **Assertion (A):** If the circumference of a circle is 176 cm, then its radius is 28 cm.
Reason (R): Circumference of circle is $2\pi \times$ radius.

a) both A and R are true and R is the correct explanation of A
b) both A and R are true and R is the not correct explanation of A
c) A is true but R is false
d) A is false but R is true

Option (a) is Correct

Solution:

Given the circumference of a circle is 176 cm

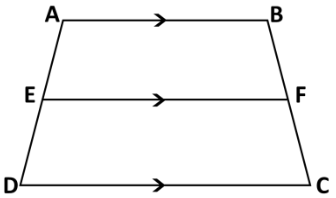
$2\pi\,r = 176 \Rightarrow 2\times\frac{22}{7}\times r = 176$

$\Rightarrow r = 28\text{cm}$

Section B

∗ This section consists of 5 questions of 2 marks each.

21. In the given figure, if ABCD is trapezium in which AB∥ CD∥ EF, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



Solution:

Given, ABCD is trapezium in which AB∥ CD∥ EF

Join AC

In Δ ABC, GF∥ AB

By Basic Proportionality Theorem

$\frac{CF}{FB} = \frac{CG}{GA}$

$\frac{BF}{FC} = \frac{GA}{CG}$ (Invertedo) -------------(1)

In Δ ADC, EG∥ DC

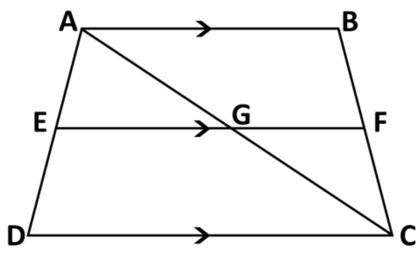
By Basic Proportionality Theorem

$\frac{AE}{ED} = \frac{GA}{CG}$ (Invertedo) ------------- (2)

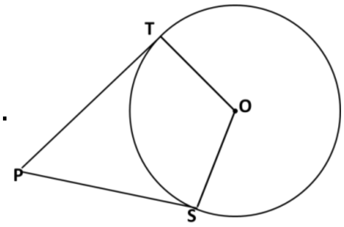
From (1) and (2)

$\frac{AE}{ED} = \frac{BF}{FC}$

Hence proved



22. In the given figure, from a point P, two tangents PT and. PS are drawn to a circle with centre O such that $\angle SPT = 120°$ Prove that $OP = 2PS$.



Solution:

It is given that PS and PT are tangents to the circle with centre O.

Also, $\angle SPT = 120°$.

To prove: $OP = 2PS$

Proof:

In $\triangle PTO$ and $\triangle PSO$,

$PT = PS$ (Tangents drawn from an external point to a circle are equal in length.)

$TO = SO$ (Radii of the circle)

$\angle PTO = \angle PSO = 90°$

$\therefore \triangle PTO \cong \triangle PSO$ (By SAS congruency)

Thus, $\angle TPO = \angle SPO = \frac{120°}{2} = 60°$

Now, In ΔPSO,

$\cos 60° = \frac{PS}{OP}$

$\frac{1}{2} = \frac{PS}{OP}$

$\Longrightarrow OP = 2PS$

Hence proved

23. If $\sqrt{3}\sin\theta - \cos\theta = 0$ where $0 < \theta < 90°$, then find the value of θ.

Solution:

Given, $\sqrt{3}\sin\theta - \cos\theta = 0$ where $0 < \theta < 90°$

$\sqrt{3}\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30^0$

$\therefore \theta = 30^0$

24. A box contains cards numbered 11 to 123. a card is drawn at random from the box find the probability that the number on the card is

(i) A perfect square number

(ii) Multiple of 7

Solution:

Total cards numbered from 11 to 123 = 113

(i) Square numbers from 11 to 123 = 16, 25, 36, 49, 64, 81, 100 and 121

Total square number from 11 to 123 = 8

P (a perfect square number) = $\frac{8}{113}$

(ii) Multiples of 7 from 11 to 123

= 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119

Total multiples of 7 from 11 to 123 = 16

P(a multiple of 7) = $\frac{16}{113}$

(OR)

If a letter of English alphabet is chosen at random, find the probability that the letter so consonant is: (i) a vowels (ii) a consonants

Solution: Total letters English alphabet =26

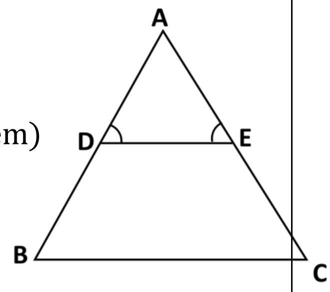
(i) no of vowels $= 5$

$P(\text{vowel}) = \frac{5}{26}$

(ii) no of consonants $= 21$

$P(\text{consonant}) = \frac{21}{26}$

25. In Figure, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$. Prove that ΔBAC is an isosceles triangle.



Solution: Given $\frac{AD}{DB} = \frac{AE}{EC}$

Therefore, DE ∥ BC (∵ Converse of Basic Proportionality Theorem)

So, $\angle D = \angle B$

$\angle E = \angle C$ (Corresponding angles) ------------ (1)

But $\angle D = \angle E$ (Given)

$\therefore \angle B = \angle C$ [From (1)]

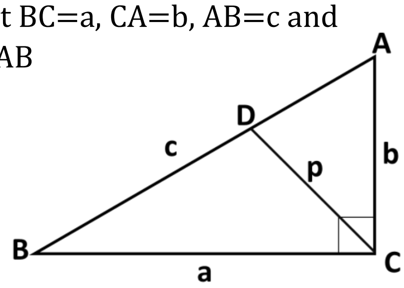
AB = AC (∵ Sides opposite to equal angles)

BAC is an isosceles triangle.

(OR)

ABC is a right triangle right angled at C. Let BC=a, CA=b, AB=c and let p be the length of perpendicular from C on AB prove that pc=ab.

Solution: Given ABC is a right triangle right angled at C. Let BC=a, CA=b, AB=c and Let p be the length of perpendicular from C on AB



In ΔACB, BC is base and AC is height

$Ar(\Delta ACB) = \frac{1}{2}\ BC \times AC$

$Ar(\Delta ACB) = \frac{1}{2}\ a \times c$ -------- (1)

Also, in ΔACB, AB is base and CD is height

$Ar(\Delta ACB) = \frac{1}{2}\ AB \times CD$

$Ar(\Delta ACB) = \frac{1}{2}\ c \times p$ -------- (2)

From (1) and (2)

pc=ab.

Section C

∗ This section consists of 6 questions of 3 marks each.

26. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Solution:

Given Polynomial is $2x^2 - 3x + 1$

$\alpha + \beta = \frac{3}{2}$; $\alpha\,\beta = \frac{1}{2}$

Quadratic polynomial whose zeroes $3\alpha, 3\beta$ is

$k\,[\,x^2 - (3\alpha + 3\beta) + (3\alpha)\,(3\beta)]$

$k\,[x^2 - 3(\alpha+\beta) + 9\alpha\beta$

$k[\,x^2 - 3\left(\frac{3}{2}\right) + 9\left(\frac{1}{2}\right)\,]$

$k\,[\,x^2 - \frac{9}{2}x + \frac{9}{2}\,]$

when $k = 2$

Quadratic polynomial is $2x^2 - 9x + 9$

27. Find whether the following pair of linear equations has a unique solution. If yes, find the solution.$7x - 4y = 49$ and $5x - 6y = 57$

Solution:

Given equations are $7x - 4y = 49$ --------------- (1)

$5x - 6y = 57$ --------------- (2)

$a_1 = 7; b_1 = -4; c_1 = 49$

$a_2 = 5; b_2 = -6; c_2 = 57$

$\frac{a_1}{a_2} = \frac{7}{5}$; $\frac{b_1}{b_2} = \frac{-4}{-6} = \frac{2}{3}$; $\frac{c_1}{c_2} = \frac{49}{57}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Given equations has a unique solution

5 equation (1) – 7 equation (2)

$$\begin{array}{l} \cancel{35x} - 25y = 245 \\ \cancel{35x} - 42y = 399 \\ \hline \quad 22y = -154 \end{array}$$

$y = -7$

Substitute y value in equation (1)

$7x - 4(-7) = 49$

$7x + 28 = 49$

$7x = 49 - 28 = 21$

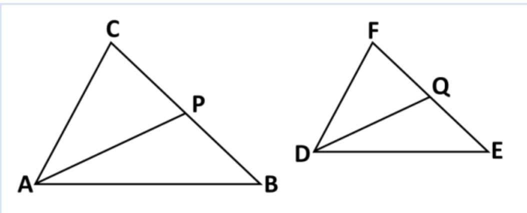
$x = 3$

$\therefore$ Solution of given equations is $x = 3$ and $y = y = -7$

28. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.

Prove that (i) $\frac{AP}{DQ} = \frac{AB}{DE}$

(ii) $\Delta CAP \sim \Delta FDQ$



Solution:

Given, $\Delta ABC \sim \Delta DEF$

AP bisects $\angle CAB$ and DQ bisects $\angle FDE$

$\Longrightarrow \angle CAP = \angle BAP$ and $\angle FDQ = \angle EDQ$

(i) In ΔABP and ΔDEQ

$\angle B = \angle E$ $(\because \Delta ABC \sim \Delta DEF)$

$\angle BAP = \angle EDQ$ $[\because \angle A = \angle D$; ½ $(\because \angle A) =$ ½ $(\angle D)]$

By A. A similarity

$\Delta ABP \sim \Delta DEQ$

$\Rightarrow \frac{AP}{DQ} = \frac{AB}{DE}$ ($\because$ corresponding sides in proportion)

(ii) In ΔCAP and ΔFDQ

$\angle C = \angle F$ ($\because \Delta ABC \sim \Delta DEF$)

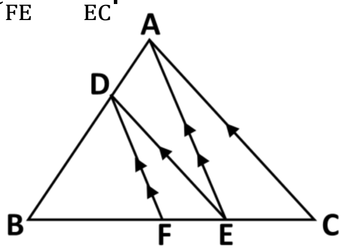
$\angle CAP = \angle FDQ$ [$\because \angle A = \angle D$; ½ ($\because \angle A$) = ½ ($\angle D$)]

By A. A similarity

$\Delta CAP \sim \Delta FDQ$

(OR)

In the following figure DE || AC and DF || AE, then prove that $\frac{BE}{FE} = \frac{BE}{EC}$.



Solution:

Given, DE || AC and DF || AE $\Delta ABC \sim \Delta DEF$

In ΔABC, DE || AC

$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC}$ ------------ (i)

($\because$ by basic proportionality theorem)

In ΔABE, DF || AE

$\Rightarrow \frac{BD}{DA} = \frac{BF}{FE}$ ------------ (ii)

($\because$ by basic proportionality theorem)

From (i) and (ii)

$\frac{BE}{FE} = \frac{BE}{EC}$

29. If cos (40 + x)= sin 30, find the value of x

Solution:

Given, cos (40 + x) = sin 30

cos (40 + x)= cos (90 – 30)

40 + x= 60

x = 60 – 40 = 20

30. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. (Use π=22/7)

Solution:

Given, Radius of a Cone = r = 5 cm

Height of the Cone = h = 24 cm

Radius of the cylinder = R = 10 cm

Height of the cylinder = H =?

Volume of water in conical vessel = Volume in cylindrical vessel

$\Rightarrow \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi R^2 H$

$\Rightarrow \frac{1}{3} \pi (5)^2 \times 24 = \pi (10)^2 \times H$

$\Rightarrow 200 = 100 \times H$

$\therefore H = 2$ cm

(OR)

504 cones, each of diameter 3.5cm and height 3cm, are melted and recast into a metallic sphere. Find the diameter of the sphere.

Solution:

Given, diameter of each cone $= 3.5$ cm

$$\Longrightarrow r = \frac{3.5}{2} \text{ cm}$$

Height of the cone $= h = 3$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi\left(\frac{3.5}{2}\right)^2 \times 3$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3$$

$$= \frac{19.25}{2}$$

Volume of 504 cones $= 504 \times \frac{19.25}{2} = 4851 \text{ cm}^3$

504 cones are melted to form a sphere.

Volume of 504 cones = volume of sphere

$\Longrightarrow$ Volume of sphere $= 4851 \text{ cm}^3$

$\frac{4}{3}\pi r^3 = 4851$

$\frac{4}{3} \times \frac{22}{7} \times r^3 = 4851$

$r^3 = \frac{4851 \times 3 \times 7}{4 \times 22}$

$r^3 = (10.5)^3$

$r = 10.5$

Thus, the diameter of sphere $= 2 \times 10.5$ cm $= 21$cm

31. Three bells toll together at intervals of 9, 12 and 15 minutes respectively . If they start tolling together, after what time will they toll next?

Solution:

Given, three bells toll together at intervals of 9, 12 and 15 minutes respectively

Tolling together for next time means

tolling after the least possible minutes which is the LCM of 9, 12 and 15

LCM of 9, 12 and 15 $= 3 \times 3 \times 5 \times 2 \times 2$

$= 180$

3	9, 12, 15
3	3, 4, 5
5	1, 4, 5
2	1, 4, 1
2	1, 2, 1
	1, 1, 1

$\therefore$ Time after which the three bells will toll together next $= 180$ minutes

We know that 60 minutes = 1 hour

$\therefore$ 180 minutes $= 180/60 = 3$ hours

Section D

***This section consists of 4 questions of 5 marks each**

32. Solve forx: $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}, x \neq 0,1,2$

Solution:

Given $\frac{1}{(x-2)} + \frac{2}{(x-1)}x = \frac{6}{x}$

$$= \frac{x-1+2x-4}{(x-2)(x-1)} = \frac{6}{x}$$
$$= x(3x-5) = 6(x-2)(x-1) = 6(x^2 - x - 2x + 2)$$
$$= 3x^2 - 5x = 6x^2 - 18x + 12$$
$$= 3x^2 - 5x = 6x^2 + -12 = 0$$
$$= 13x - 3x^2 + 12 = 0$$
$$= 3x^2 - 13x + 12 = 0$$
$$= 3x^2 - 4x - 9x + 12 = 0$$
$$= x(3x-4) - 3(3x-4) = 0$$
$$= (3x-4)(x-3) = 0$$
$$x = 3 \text{ or } x = \frac{4}{3}$$

(OR)

find the values of k for which the quadratic equation $(3k + 1)x^2 + 2(k + 1)\,x + 1 = 0$ has equal roots. Also, find the roots.

Solution:

Given $y = (3k+1)x^2 + 2(k+1)x + 1 = 0$

Then $D = 0 \Rightarrow b^2 - 4ac = 0$

$a = 3k + 1b = 2(k+1)c = 1$

$b^2 - 4ac = [2(k+1)]^2 - 4(3k+1) = 0$

$\Rightarrow 4k^2 + 4 + 8k - 12k - 4 = 0$

$\Rightarrow 4k^2 - 4k = 0$

$\Rightarrow 4k(k-1) = 0$

$\Rightarrow k = 0k = 1$

When $k = 0$

equation $y = x^2 + 2x + 1 = 0$

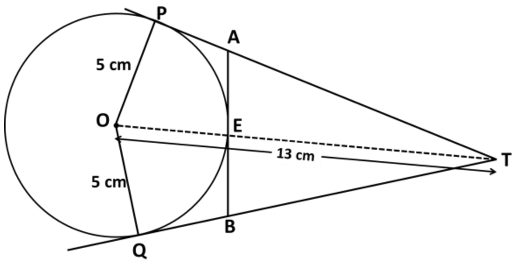
Roots are $x = -1, -1$

When $k = 1$

equation $4x^2 + 4x + 1 = 0$

Roots are $x = \frac{-1}{2}, \frac{-1}{2}$

33. O is the centre of a circle of radius 5cm. T is a point such that OT=13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find length of AB.



Solution:

Clearly $\angle OPT = 90^0$

Applying Pythagoras in $\triangle OPT$, we have

$OT^2 = OP^2 + PT^2$

$\Rightarrow 13^2 = 5^2 + PT^2$

$\Rightarrow PT^2 = 169 - 25 = 144$

$\Rightarrow PT = 12$ cm

Since lengths of tangents drawn from a point to a circle are equal.

Therefore, AP=AE=x (say)

$\Rightarrow AT = PT - AP = (12 - x)$ cm

Since, AB is the tangent to the circle E.
Therefore, OE⊥AB
$\Rightarrow \angle OEA = 90^0$
$\Rightarrow \angle AET = 90^0$
$\Rightarrow AT^2 = AE^2 + ET^2$ [Applying pythagoras theorem in △AET]
$(12 - x)^2 = x^2 + (13-5)^2$
$\Rightarrow 144 - 24x + x^2 = x^2 + 64$
$\Rightarrow 24x = 80$
$\Rightarrow x = \frac{10}{3}$cm
Similarly, BE=$\frac{10}{3}$cm
$\therefore$ AB=AE+BE= $(\frac{10}{3}+\frac{10}{3})$cm
$= \frac{20}{3}$cm

34. Find the mode of the following frequency distribution

Class Interval	25 – 30	30 –35	35 – 40	40 – 45	45 – 50	50 – 55
Frequency	25	34	50	42	38	14

Solution:

35 – 40 is model class (highest frequency)
l = lower limit = 35
h = height (class width) = 5
f_0 = Frequency of preceding class = 34
f_1 = Frequency of modal class = 50
f_2 = Frequency of succeeding class = 42

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$
$$= 35 + \frac{50-34}{2(50)-34-42} \times 5$$
$$= 35 + \frac{16}{100-76} \times 5$$
$$= 35 + \frac{16}{24} \times 5$$
$$= 35 + \frac{80}{24}$$
$$= 35 + 3.33$$

$\therefore$ Mode is 38.33

(OR)

On the sports day of a school, 300 students participated. Their ages are given in the following distribution.

Age (in years)	5 – 7	7 – 9	9 – 11	11– 13	13 – 15	15 – 17	17 – 19
Number of students	67	33	41	95	36	13	15

Find the mean and mode of the date.

Solution:

C.I	f_i	x_i	$f_i\,x_i$
5−7	67	6	402
7−9	33	8	264
9−11	41	10	410
11−13	95	12	1140
13−15	36	14	504
15−17	13	16	208
17−19	15	18	270
	$\sum f_i = 300$		$\sum f_i\,x_i = 3198$

$$\text{Mean} = \frac{\sum f_i\,x_i}{\sum f_i}$$
$$= \frac{3198}{300}$$
$$= 10.66$$

11 − 13 is model class (highest frequency)

l = lower limit = 11

h = height (class width) = 2

f_0 = Frequency of preceding class = 41

f_1 = Frequency of modal class = 95

f_2 = Frequency of succeeding class = 36

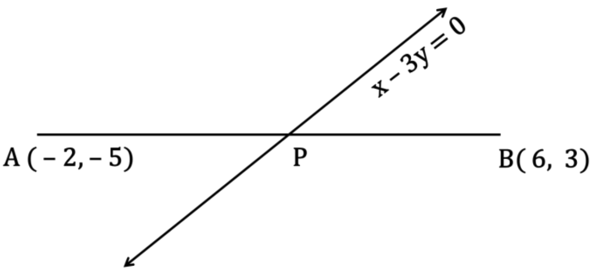
$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$
$$= 11 + \frac{95 - 41}{2(95) - 41 - 36} \times 2$$
$$= 11 + \frac{54}{190 - 77} \times 2$$
$$= 11 + \frac{54}{113} \times 2$$
$$= 11 + \frac{108}{113}$$
$$= 11 + 0.95$$

$\therefore$ Mode is 11.95

35. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the coordinates of the point of intersection.

Solution:

Let the line $x - 3y = 0$ intersect the segment joining the points A $(\overset{x_1}{-2}, \overset{y_1}{-5})$ and B$(\overset{x_2}{6}, \overset{y_2}{3})$ at P in the ratio $\overset{m}{k} : \overset{n}{1}$.



$$P = \left(\frac{m\,x_2 + n x_1}{m+1}, \frac{m\,y_2 + n\,y_1}{m+1}\right)$$
$$P = \left(\frac{k\,(6) + 1(-2)}{k+1}, \frac{k\,(3) + 1(-5)}{k+1}\right)$$
$$P = \left(\frac{6k - 2}{k+1}, \frac{3k - 5}{k+1}\right)$$

But P lies on the line $x - 3y = 0$

$$\frac{6k - 2}{k+1} - 3\left(\frac{3k - 5}{k+1}\right) = 0$$
$$\frac{(6k - 2) - 3(3k - 5)}{k+1} = 0$$
$$6k - 2 - 9k + 15 = 0$$

$-3k + 13 = 0$

$-3k = -13$

$k = \frac{13}{3}$

$\therefore$ Ratio is 13 : 3

$\Rightarrow$ Coordinates of P are $(\frac{9}{2}, \frac{3}{2})$

Section E

***This section consists of 3 questions of 4 marks each**

36. Case Study – 1

Volume of a Bird Cage. A company makes rectangular shaped bird cages with height b inches and square bottoms. The volume of these cages is given by the function $V = b^3 - 6b^2 + 9b$.

(i) Find an expression for the length of each side of the square bottom.
(ii) Use the function to find the volume of a cage with a height of 18 inches.
(iii) Use the remainder theorem to find the volume of a cage with a height of 15 inches.
(iv) Verify the result of (iii) using functions.

Solution:

(i) $V = b^3 - 6b^2 + 9b$

$= b(b^2 - 6b + 9)$

$= b[b^2 - 3b - 3b + 9]$

$= b(b-3)(b-3)$

$\Rightarrow$ b is height

$(b-3)$ is the length of each side of square bottom.

(ii) If b = 18 inches (height of cage)

Volume of cage $= b(b-3)(b-3)$

$= 18(18-3)(18-3)$

$= 18 \times 15 \times 15$

$= 4050 \text{ (inch)}^2$

(iii) Here we have to find V (15) using the remainder theorem.

We divide $b^3 - 6b^2 + 9b$ by $b - 15$. Remainder will be value of V (15).

$$\begin{array}{r|l} & b^2 + 9b + 144 \\ \hline b - 15 & b^3 - 6b^2 + 9b \\ & b^3 - 15b^2 \\ \hline & \quad 9b^2 + 9b \\ & \quad 9b^2 - 135b \\ \hline & \qquad 144b \\ & \qquad 144b - 2160 \\ \hline & \qquad\qquad 2160 \end{array}$$

Remainder is 2160

$V(15) = 2160$

(iv) $V(15) = 15(15-3)(15-3)$

$= 15 \times 12 \times 12$

$= 2160 \text{ (inch)}^2$

37. Case Study – 2

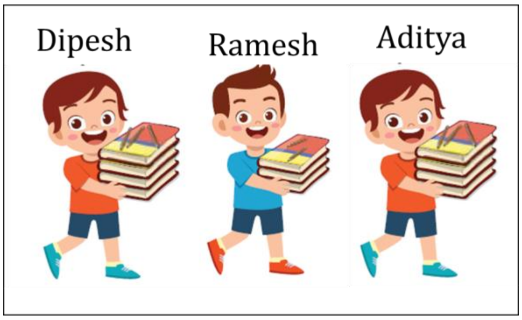
Dipesh bought 3 notebooks and 2 pens for Rs. 80. His friend Ramesh said that price of each notebook could be Rs. 25. Then three notebooks would cost Rs.75, the two pens would cost Rs. 5 and each pen could be for Rs. 2.50.Another friend Amar felt that Rs. 2.50 for one pen was too little. It should be at least Rs. 16. Then the price of each notebook would also be Rs.16. Aditya also bought the same types of notebooks and pens as Dipesh. He paid 110 for 4 notebooks and 3 pens.

(i) Whether the estimation of Ramesh and Amar is applicable for Aditya?

(ii) Let the cost of one notebook be x and that of pen be y . Which of the following set describe the given problem?

(iii) What is the exact cost of the notebook?

(iv) What is the exact cost of the pen? What is the total cost if they purchase the same type of 15 notebooks and 12 pens?



Solution:

(i) Let cost of one notebook be x and pen be y

Then cost of 3 notebooks and 2 pens = $3x + 2y$

Again cost of 4 notebooks and 3 pens = $4x + 3y$

So by the given condition

$3x + 2y = 80$ .... (1)

$4x + 3y = 110$ .... (2)

Which is set of equations to describe the given problem?

Hence the set describe the given problem is

$3x + 2y = 80$ and $4x + 3y = 110$

Now we solve from x and y

Multiplying Equation (1) by 3 and equation (2) by 2 we get

$9x + 6y = 240$

$8x + 6y = 220$

On subtraction we get $x = 20$

From Equation (1) we get $y = 10$

Hence cost of one notebook is 20 and that of pen be 10

Ramesh said that price of each notebook could be Rs. 25

So Ramesh's estimation is wrong

Amar felt that Rs. 2.50 for one pen was too little. It should be at least Rs. 16

So Amar's estimation is wrong

(ii) Whether the estimation of Ramesh and Amar is applicable for Lokesh

Ramesh's estimation is wrong but Amar's estimation is also wrong.

Solving Equation (1) & Equation (2)

we get $x = 20$ & $y = 10$

(iii) The exact cost of the notebook = Rs 20

(iv) The exact cost of the pen = Rs 10

The cost of 15 notebooks = Rs (15×20)

= Rs 300

The cost of 12 pens = Rs (12×10)

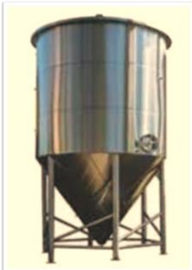
= Rs 120

Hence total cost = Rs ($300 + 120$)

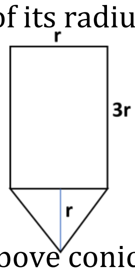
= Rs 420

38. Case Study – 3

Conical Tank : The advantages of cone bottom tanks are found in nearly every industry, especially where getting every last drop from the tank is important. This type of tank has excellent geometry for draining, especially with high solids content slurries as these cone tanks provide a better full-drain solution. The conical tank eliminates many of the problems that flat base tanks have as the base of the tank is sloped towards the centre giving the greatest possible full-drain system in vertical tank design.



Rajesh has been given the task of designing a conical bottom tank for his client. Height of conical part is equal to its radius. Length of cylindrical part is the 3 times of its radius. Tank is closed from top. The cross section of conical tank is given below.



(i) If radius of cylindrical pait is taken as 3 meter, what is the volume of above conical tank ?

(ii) What is the area of metal sheet used to make this conical tank ? Assume that tank is covered from top.

(iii) What is the ratio of volume of cylindrical part to the volume of conical part?

(iv) The cost of metal sheet is Rs 2000 per square meter and fabrication cost is 1000 per square meter. What is the total cost of tank?

Solution:

(i) Radius of the cylindrical part = Radius of the conical part = r = 3 m

Height of the cylindrical part = $3r = 3 \times 3 = 9$ m

Height of the conical part = r = 3 m

$\therefore$ The volume of the conical bottomed tank

= Volume of cylinder + Volume of cone

$= \pi r^2 h + \frac{1}{3} \pi r^2 h$

$= \pi (3)^2 \times 9 + \frac{1}{3} \pi (3)^2 \times 3$

$= 81\pi + 9\pi$

$= 90\pi$ m^3

(ii) Finding the area of metal sheet used to make this conical tank:

Radius, $r = 3$ m

$\therefore$ Slant height, $l = \sqrt{r^2 + h^2}$

$l = \sqrt{3^2 + 3^2}$

$l = \sqrt{18}$

$l = 3\sqrt{2}$ m

$\therefore$ The area of the metal sheet used to make this conical tank is

= [CSA of the cylindrical part] + [Area of the top of the cylindrical part] + [CSA of the conical part]

$= 2\pi rh + \pi r^2 + \pi r l$

$= 2 \times \pi \times 3 \times 9 + \pi 3^2 + \pi \times 3 \times 3\sqrt{2}$

$= 54\pi + 9\pi + 9\sqrt{2}\pi$

$= 63\pi + 9\sqrt{2}\pi$

$= 9\pi (7 + \sqrt{2})$ m^2

(iii) Finding the ratio of the volume of the cylindrical part to the volume of the conical part:

$\therefore$ the ratio of the volume of the cylindrical part to the volume of the conical part,

$= \pi r^2 h_1 : \frac{1}{3} \pi r^2 h_2$

$= h_1 : \frac{1}{3} h_2$

$= 9 : \frac{1}{3} \times 3$

$= 9 : 1$

(iv) Finding the total cost of the tank:

The cost of the metal sheet is = Rs 2000 per square meter

The fabrication cost is = Rs. 1000 per square meter

$\therefore$ The total cost of the tank per meter square is = Rs. 2000 + Rs. 1000 = Rs. 3000

$\therefore$ The total cost of the conical bottom tank is $= 3000 \times 9\pi (7 + \sqrt{2})$

$= 27000\pi (7 + \sqrt{2})$

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