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CBSE

X CLASS

Mathematics

SOLVED QUESTION PAPERS

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Section A

※This section consists of 20 MCQ'S questions of 1 mark each

1. The distance between the points $(a\cos\theta + b\sin\theta, 0)$ and $(0, a\sin\theta - b\ co\ \theta)$ is ____.

a) $a^2 + b^2$ b) $a^2 - b^2$ c) $\sqrt{a^2+b^2}$ d) $\sqrt{a^2-b^2}$

Option (c) is Correct

Solution: Given the point $A(\cos\theta + b\sin\theta, 0), (o, a\sin\theta - b\cos\theta)$

By distance formula, The distance of

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{[0-(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta) - o]^2}$$
$$= \sqrt{a^2\cos^2\theta + 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta}$$
$$= \sqrt{(a^2+b^2)\cos^2\theta + (a^2+b^2)\sin^2\theta} = \sqrt{a^2+b^2}$$
$$[\because \cos^2\theta + \sin^2\theta = 1]$$

2. If one zero of the polynomial $(3x^2 + 8x + k)$ is the reciprocal of the other, then value of k is

a) 3 b) -3 c) $\frac{1}{3}$ d) $-\frac{1}{3}$

Option (a) is Correct

Solution: $p(x) = 3x^2 + 8x + k$

By the sum, let the zero be a.

$$\text{then, } a = \frac{1}{a}$$
$$\text{here; } a = 3, b = 8, c = k$$
$$\text{so, } a \times \frac{1}{a} = \frac{k}{3}$$
$$1 = \frac{k}{3}$$
$$k = 3$$

3. If $3x + 4y : x + 2y = 9 : 4$, then $3x + 5y : 3x - y$ is equal to

a) 4: 1 b) 1: 4 c) 7: 1 d) 1: 7

Option (c) is Correct

Solution: Given $3x + 4y : x + 2y = 9 : 4$

$$\frac{3x+4y}{x+2y} = \frac{9}{4}$$
$$\Rightarrow 9(x+2y) = 4(3x+4y)$$
$$\Rightarrow 9x + 18y = 12x + 16y$$
$$\Rightarrow 3x = 2y$$

Now $3x + 5y : 3x - y =$

$$\frac{3x+5y}{3x-y} = \frac{2y+5y}{2y-y} = \frac{7y}{y}$$
$$= 7 : 1$$

4. The value of 'k' for which the system of equations $x + 2y = 3$ and $5x + ky + 7 = 0$ inconsistent is

a) $-\frac{14}{3}$ b) $\frac{2}{5}$ c) 5 d) 10

Option (d) is Correct

Solution: Given equations $x + 2y = 3$ and $5x + ky + 7 = 0$ are inconsistent

$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\Rightarrow \frac{1}{5} = \frac{2}{k}$

$\Rightarrow k = 5 \times 2 = 10$

5. If α and β are the zeroes of the polynomial $2x^2 - 13x + 6$, then α + β is equal to

a) – 3 b) 3 c) $\frac{13}{2}$ d) $-\frac{13}{2}$

Option (c) is Correct

Solution: Given α and Β are the zeroes of the polynomial $2x^2 - 13x + 6$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-13}{2} = \frac{13}{2}$$

6. The roots of the quadratic equation $x^2 - 0.04 = 0$ are

a) ± 0.2 b) ±0.02 c) 0.4 d) 2

Option (a) is Correct

Solution: Given equation is $x^2 - 0.04 = 0$

$x^2 = 0.04$

$x = \sqrt{0.04} = 0.2$

7. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

a) 5 b) 20 c) 25 d) 30

Option (c) is Correct

Solution: Given common difference of an AP is 5

$a_{18} - a_{13} = a + 17d - (a + 12d)$

$= a + 17d - a - 12d$

$= 17d - 12d$

$= 5d$

Here d = 5

$a_{18} - a_{13} = 5(5) = 25$

8. ABC is an equilateral triangle with each side of length 2p. If AD ⊥ BC then value of AD is

a) $\sqrt{3}$ b) $\sqrt{3}\,p$ c) 2p d) 4p

Option (b) is Correct

Solution: Given ABC is an equilateral triangle with each side of length 2p

In Δ ABC, AB = BC = AC = 2p

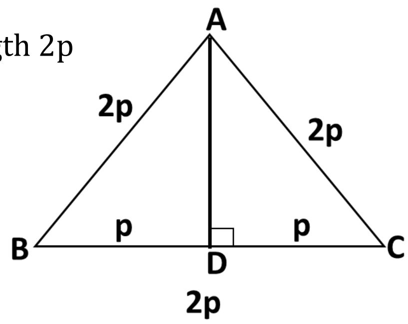
Now in Δ ADB, $AB^2 = AD^2 + BD^2$

$(2p)^2 = AD^2 + (p)^2$

$4p^2 = AD^2 + p^2$

$3p^2 = AD^2$

$AD^2 = \sqrt{3p^2} = \sqrt{3}\,p$



9. The base radii of a cone and a cylinder are equal. If their curved surface areas are also equal, then the ratio of the slant height of the cone to the height of the cylinder is

a) 2 : 1 b) 1 : 2 c) 1 : 3 d) 3 : 1

Option (a) is Correct

Solution: Let the radius of the cone and cylinder be r.

The base radii of cone and cylinder are equal

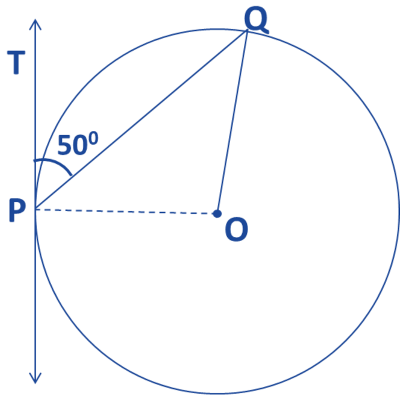
Given curved surface areas are equal,

$\therefore \pi r\, l = 2\pi r\, h$

$\frac{l}{h} = 2$ $\therefore l : h = 2 : 1$

10. In figure, O is the center of circle. PQ is a chord and PT is tangent at P which makes an angle of 50^0 with PQ, then $\angle POQ$ is

a) 130^0 b) 90^0 c) 100^0 d) 75^0



Option (c) is Correct

Solution:

Since OP is perpendicular to PT

$\angle OPT = 90^0$

$\angle OPQ + \angle QPT = 90^0$

$\angle OPQ + 50° = 90^0$

$\angle OPQ = \angle OQP = 40^0$

($\because$ OP = OQ = radii; so Isosceles triangle OPQ)

Again,

$\angle POQ + \angle OPQ + \angle OQP = 180^0$

($\because$ Angle sum property of triangle)

$\angle POQ + 40^0 + 40^0 = 180^0$

$\angle POQ = 180^0 - 40^0 - 40^0 = 100°$

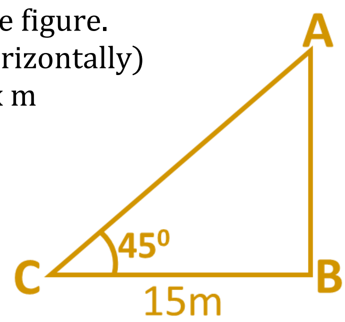
11. A tree casts a shadow 15 m long on the level of ground, when the angle of elevation of the sun is 45c. The height of a tree is

B a) 10 m b) 14 m c) 8 m d) 15 m

Option (d) is Correct

Solution:

So, Lets consider a right-angled triangle ABC as shown in the figure.



In ΔABC, Base BC = 15 m (as the shadow is at the ground horizontally)

We have to find the height of the tree i.e. height of Δ ABC = x m

Now, $\angle C = 45°$

$\tan 45° = \frac{AB}{BC}$

$1 = \frac{AB}{15}$

AB = 15m

$\therefore$ The height of the tree = 15 m

12. The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has

a) Two distinct real roots b) Two equal real roots

c) No real roots (d) More than 2 real roots

Option (c) is Correct

Solution: Given equation is $2x^2 - \sqrt{5}\,x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$

we get $a=2, b=-\sqrt{5}$ and $c=1$

$D = b^2 - 4ac = (-\sqrt{5})^2 - 4\,(2)(1) = 5 - 8 - 3 < 0$

Discriminant $= -3 < 0$

Since, discriminant is negative

$\therefore$ Quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has no real roots

13. A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being 240^0. If another circle of the area same as the sector is formed, then radius of the new circle is

a) 79.5 cm b) 81.6 cm c) 83.4 cm d) 88.5 cm

Option (b) is Correct

Solution

Area of sector $= \frac{240}{360} \times \pi\ (100)^2 = \frac{20000}{3}\ \pi\ cm^2$

Let r be the radius of the new circle, then

$\pi r^2 = \frac{20000}{3}\ \pi$

$r^2 = \frac{20000}{3}$

$r^2 = 6666.67$

$r = \sqrt{6666.67} = 81.6496$

$r = 81.6$ (Approx.)

14 In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is

a) 6 b) 7 c) 8 d) 12

Option (b) is Correct

Solution:

The mid value of the class $=10$

Width of the interval $=6$

then lower limit = Mid value$-\frac{\text{width}}{2} = 10-3=7$

15. If a card is selected from a deck of 52 cards, then the probability of its being a red face card is

a) $\frac{3}{26}$ b) $\frac{3}{13}$ c) $\frac{2}{13}$ d) $\frac{1}{2}$

Option (a) is Correct

Solution:

There are 52 cards in a deck of playing cards. If a card is drawn from this well-shuffled deck, the total number of all possible outcomes is 52

$n(S)=52$

Let A be the event of drawing a red face card.

Number of face cards in the deck is 12

Number of red face cards in the deck is A=6

$P\ (A) = \frac{6}{52} = \frac{3}{26}$

16. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

a) $\cos\beta$ b) $\cos 2\beta$ c) $\sin\alpha$ d) $\sin 2\alpha$

Option (b) is Correct

Solution:

$\cos(\alpha+\beta)=0$

$\Rightarrow \cos(\alpha+\beta) = \cos 90\ (\because \cos 90 = 0).$

$\Rightarrow \alpha + \beta = 90$

$\Rightarrow \alpha = 90-\beta$

$\sin(\alpha-\beta) = \sin(90-2\beta) = \cos 2\beta\ \ (\because \sin(90-\theta) = \cos\theta)$

17. The point P on X – axis equidistant from the points A $(-1, 0)$ and B $(5, 0)$ is

a) $(2, 0)$ b) $(0, 2)$ c) $(3, 0)$ d) $(-3, 5)$

Option (a) is Correct

Solution:

Let $(-1, 0)$ and $(5, 0)$ be equidistant from P $(x, 0)$

AP = BP

$\Longrightarrow AP^2 = BP^2$

$(x + 1)^2 + 0 = (x - 5)^2 + 0$
$\Longrightarrow x^2 + 2x + 1 = x^2 - 10x + 25$
$\Longrightarrow 12x = 24$
$\Longrightarrow x = 2$
Hence P is (2, 0)

18. The point on the X – axis which is equidistant from the points A (−2, 3) and B(5, 4) is
a) (0, 2) b) (2, 0) c) (3, 0) d) (− 2, 0)

Option (b) is Correct

Solution:

Let P(x, 0) be a point on X-axis such that
AP=BP
$\Longrightarrow AP^2 = BP^2$
$\Longrightarrow (x+2)^2 + (0-3)^2 = (x-5)^2 + (0-4)^2$
$\Longrightarrow x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$
$\Longrightarrow 14x = 28$
$\Longrightarrow x = 2$
∴ P (2, 0) is required point on X – axis

☞ **In the question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correction option.**

19. **Assertion:** When a positive integer a is divided by 3, the values of remainder can be 0, 1 or 2.
Reason: According to Euclid's Division Lemma $a = bq + r$, where $0 \le r < b$ and r is an integer.
a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
c) Assertion (A) is true but reasons (R) is false.
d) Assertion (A) is false but reasons (R) is true.

Option (a) is Correct

Solution:

Euclid's division Lemma:
It tells us about the divisibility of integers. It states that any positive integer 'a' can be divided by any other positive integer ' b' in such a way that it leaves a remainder 'r'.
Euclid's division Lemma states that for any two positive integers 'a' and 'b' there exist two unique whole numbers 'q' and 'r' such that , $a = bq + r$, where $0 \le r < b$.
Here, a= Dividend, b= Divisor, q= quotient and r = Remainder.
According to Euclid's division lemma
$a = 3q + r$, where $0 \le r < 3$.
∴ the values of r can be 0, 1 or 2

20. **Assertion:** Sum of first 10 terms of the arithmetic progression −0.5, −1.0, −1.5 ... is 31.
Reason: Sum of n terms of an AP is given as $S_n = \frac{n}{2}[2a + (n-1)d]$ where a is first term and d common Difference.

a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
c) Assertion (A) is true but reason (R) is false.
d) Assertion (A) is false but reason (R) is true.

Option (d) is Correct

Solution:

Given Sum of first 10 terms of the arithmetic progression $-0.5, -1.0, -1.5 \ldots$ is 31

$S_{10} = \frac{10}{2}[2(-0.5) + 9(-0.5)]$

$= 5[-1 - 4.5]$

$= 5(-5.5)$

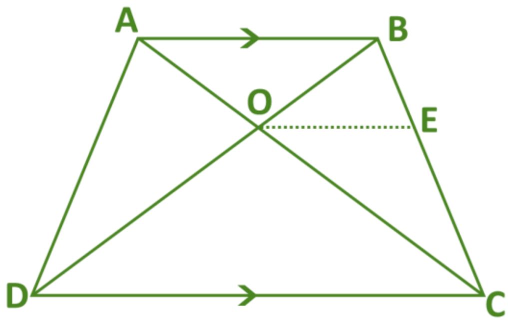
$= 27.5$

Section B

∗ This section consists of 5 questions of 2 marks each.

21. ABCD is a trapezium in which AB | | CD and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Solution:



Given, ABCD is trapezium in which AB∥ CD

Draw OE∥DC such that E lies on BC

In Δ BDC, OE∥ DC

By Basic Proportionality Theorem

$\frac{BO}{DO} = \frac{BE}{EC}$ ------------- (1)

In Δ ABC, OE ∥ AB

By Basic Proportionality Theorem

$\frac{CO}{AO} = \frac{EC}{BE}$

$\frac{AO}{CO} = \frac{BE}{EC}$ (Invertedo) ------------- (2)

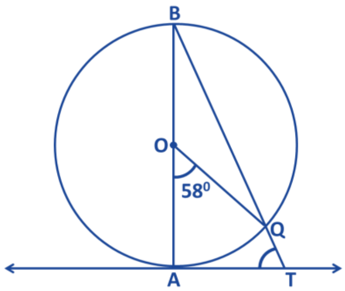
From (1) and (2)

$\frac{AO}{BO} = \frac{CO}{DO}$

Hence proved

22. In given figure, AB is the diameter of a circle with center O and AT is a tangent. If $\angle AOQ = 58^0$, find $\angle ATQ$.

Solution:



$\angle ABQ = \frac{1}{2}\angle AOQ$

$= \frac{1}{2} \times 58^0 = 29^0$

$\angle OAT = 90^0$ (AT is a tangent)

$\Rightarrow \angle BAT = 90^0$

In Δ ABT

$\angle BAT + \angle ABT + \angle ATB = 180^0$ (∵ angle sum property of triangle)

$90^0 + 29^0 + \angle ATB = 180^0$

$\angle ATB = 180^0 - 119^0$

$\therefore \angle ATQ = 61^0$[∵ B, Q and T are collinear points]

23. Find the value of $\cos 2\theta$, if $2\sin 2\theta = \sqrt{3}$

Solution:

Given, $2\sin 2\theta = \sqrt{3}$

$\sin 2\theta = \frac{\sqrt{3}}{2}$

$\sin 2\theta = \sin 60^0$

$2\theta = \sin 60^0$

$\therefore \theta = 30^0$

Now $\cos 2\theta = \cos 2(30^0)$

$= \cos 60^0$

$= \frac{1}{2}$

24. Find the mean of the following distribution:

Class	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
frequency	2	3	7	6	6	6

Solution:

Class interval	Mid values (x_i)	Frequency (f_i)	$d = x_i - A$	$f_i d_i$
10-25	17.5	2	-30	– 60
25-40	32.5	3	-15	– 45
40-55	47.5=A	7	0	0
55-70	62.5	6	15	90
70-85	77.5	6	30	180
85-100	92.5	6	45	270
		$\sum f_i = 30$		$\sum f_i d_i = 435$

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i} = 47.5 + \frac{435}{30}$$

$$= 47.5 + 14.5 \quad = 62$$

(OR)

Find the mean of the following data :

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100– 120
Frequency	20	35	52	44	38	31

Solution:

Class interval	Mid values (x_i)	Frequency (f_i)	$d = x_i - A$	$f_i d_i$
0 – 20	10	20	-40	– 800
20 – 40	30	35	-20	– 700
40 – 60	50=A	52	0	0
60 – 80	70	44	20	880
80 – 100	90	38	40	1520
100– 120	110	31	60	1860
		$\sum f_i = 220$		$\sum f_i d_i = 2760$

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i} = 50 + \frac{2760}{220}$$

$$= 50 + 12.55 \quad = 62.55$$

25. Show that $5\sqrt{6}$ is an irrational number

Solution:

Let us assume that $5\sqrt{6}$ be a rational number

$\Rightarrow 5\sqrt{6} = \frac{a}{b}$ where a and b are integers and $b \neq 0$

$\sqrt{6} = \frac{a}{5b}$

Since, a, b are integers $\frac{a}{5b}$ is a rational number and also $\sqrt{6}$ is a rational number

This is a contradiction to the fact that $\sqrt{6}$ is an irrational number

Our assumption is wrong.

$\therefore 5\sqrt{6}$ is an irrational number.

(OR)

Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Solution:

$\sqrt{2} = 1.4142...$

$\sqrt{3} = 1.732.....$

Any terminating decimal between 1.4142 and 1.732 will be a rational number like, 1.5

Section C

∗ This section consists of 6 questions of 3 marks each.

26. Which term of the AP 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$ ……… is the first negative term

Solution:

Given, A.P. is 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$ ………

$= 20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, ...$

Here, $a = 20$, $d = \frac{77}{4} - 20$

$= \frac{77-80}{4} = -\frac{3}{4}$

Let a_n is first negative term

$\Rightarrow a + (n-1)\,d < 0$

$\Rightarrow 20 + (n-1)\left(-\frac{3}{4}\right) < 0$

$\Rightarrow 20 - \frac{3n}{4} + \frac{3}{4} < 0$

$\Rightarrow 20 + \frac{3}{4} < \frac{3n}{4}$

$\Rightarrow \frac{4(20) + 3}{4} < \frac{3n}{4}$

$\Rightarrow \frac{83}{4} < \frac{3n}{4}$

$\Rightarrow \frac{83}{3} < n$

$\Rightarrow n > \frac{83}{3} = 27.66$

$\therefore 28^{th}$ term will be the first negative term of given A.P.

27. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\frac{1}{2}$.

Solution:

Given $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing on both sides by $\cos^2\theta$

$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{3\sin\theta\cos\theta}{\cos^2\theta}$

$\sec^2\theta + \tan^2\theta = 3\tan\theta$

$1 + \tan^2\theta + \tan^2\theta = 3\tan\theta$

$1 + 2\tan^2\theta = 3\tan\theta$

$2\tan^2\theta - 3\tan\theta + 1 = 0$

$2\tan^2\theta - 2\tan\theta - \tan\theta + 1 = 0$

$2\tan\theta(\tan\theta - 1) - 1(\tan\theta - 1) = 0$

$(\tan\theta - 1)(2\tan\theta - 1) = 0$

$\tan\theta - 1 = 0$ or $2\tan\theta - 1 = 0$

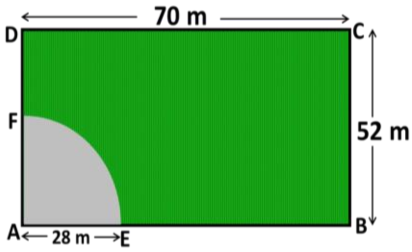
$\tan\theta = 1$ or $2\tan\theta = 1 \ \frac{1}{2}$

$\tan\theta = 1$ or $\tan\theta = \frac{1}{2}$

Hence proved

28. A horse is tethered to one corner of a rectangular field of dimensions 70 m × 52m, by a rope of length 21 m. How much area of the field can it graze?

Solution:



Length of the rectangle, l =70 m

Breadth of the rectangle, b =52 m

Length of the rope =28 m

Shaded portion AEF indicates the area in which the horse can graze.

Clearly, it is the area of a quadrant of a circle of radius, r =28 m

Area of the quadrant AEF $= \frac{1}{4} \times \pi r^2$ sq. units

$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

$= 616 m^2$

∴ Grazing Area = $616 m^2$

Area left unglazed = Area of the rectangle ABCD - Area of the quadrant AEF

Area of the rectangle ABCD = l × b sq. units

$= 70 \times 52$

$= 3640 m^2$

∴ Area left unglazed $= 3640 - 616$

$= 3024\ m^2$

(OR)

In the given figure, a chord AB of the circle with center O and radius 10 cm, that subtends a right angle at the center of the circle. Find the area of the minor segment AQBP. Hence find the area of major segment ALBQA. (Use π = 3.14).

Solution:

∠ALB=12∠AOB=90 2=450

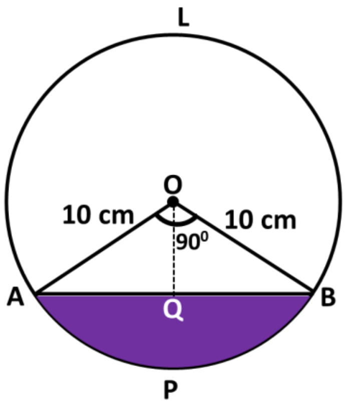
LQ is perpendicular bisection on AB.

Hence by isosceles triangles property

LA=LB

OB=10cm, & OA=10cm

AB=√102+102=10√2cm
QB=AB2=5√2cm
OQ=√OB2−QB2=√100−50=√50=5√2cm
LQ=LO+OQ
=(10+5√2)cm
Area of ALBQA=12×AB×LQ
=12×10√2×(10+5√2)
=50(1+√2)cm
2Area of AQBPA=πr24−Ar.ofAOB
=π×1024−12×102
⇒28.54cm2



29. Find the mode of the following frequency distribution:

Class	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40– 45
Frequency	3	8	9	10	3	2

Solution:

Class interval	Frequency
15 – 20	3
20 – 25	8
l 25 – 30	9 f_0
30 – 35	10 f_1
35 – 40	3 f_2
40 – 45	2

Modal class

(30-35) is modal class since it has highest frequency

L=30, h=5, f_0=9, f_1=10, f_3=3

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 30 + \frac{10-9}{2(10)-9-3} \times 5$$

$$=30 + \frac{1}{20-12} \times 5$$

$$=30 + \frac{1}{8} \times 5$$

$$=30 + \frac{5}{8}$$

$$=30 + 0.625$$

∴ Mode is 30.625

30. Find the ratio in which the segment joining the points (1, − 3) and (4, 5) is divided by X - axis? Also find the coordinates of this point on X -axis.

Solution:

Let P (x, 0) any point on the X - axis

7Given A (1, − 3) and B (4, 5)

Let P divides the line joining the points (1, − 3) and (4, 5) in the ratio k : 1

$$P = \left(\frac{k x_2 + x_1}{k+1}, \frac{k y_2 + y_1}{k+1}\right)$$

$$(x, 0) = \left(\frac{4k+1}{k+1}, \frac{5k-3}{k+1}\right)$$

By comparing both the sides, we get

$\frac{5k-3}{k+1} = 0$

$5k - 3 = 0$

$\Rightarrow 5k = 3$

$= \frac{3}{5}$

$k : 1 = 3:5$

$\therefore$ P divides the line segment joining two points in the ratio 3:5 internally

Now $x = \frac{4\left(\frac{3}{5}\right) + 1}{\frac{3}{5} + 1}$

$x = \frac{12+5}{3+5}$

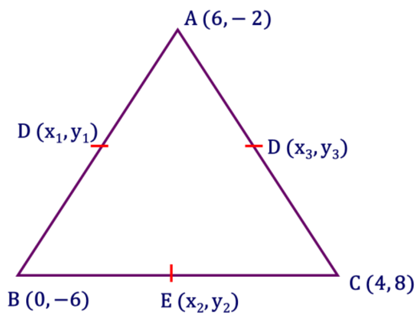
$x = \frac{17}{8}$

$\therefore$ The coordinates of this point on X –axis $= \left(\frac{17}{8}, 0\right)$

(OR)

The vertices of ΔABC are A $(6, -2)$, B $(0, -6)$ and C $(4, 8)$. Find the co-ordinates of mid-points of AB, BC and AC.

Solution:



Let the mid-points of AB, BC and CA be D (x_1, y_1), E (x_2, y_2), F(x_3, y_3),

D (x_1, y_1) is the midpoint of AB

D $(x_1, y_1) = \left(\frac{6+0}{2}, \frac{-2-6}{2}\right) = (3, -4)$

E (x_2, y_2) is the midpoint of BC

E $(x_2, y_2) = \left(\frac{0+4}{2}, \frac{-6+8}{2}\right) = (2, 1)$

F(x_3, y_3) is the midpoint of CA

F$(x_3, y_3) = \left(\frac{4+6}{2}, \frac{-2+8}{2}\right) = (5, 3)$

31. Write the smallest number which is divisible by both 306 and 657.

Solution:

To find the smallest number divisible by both 306and 657 we need to find the L.C.M.

Prime factors of $306 = 2 \times 3 \times 3 \times 17 = 2 \times 3^2 \times 17$

Prime factors of $657 = 3 \times 3 \times 73 = 3^2 \times 73$

LCM $(306,657) = 2 \times 3^2 \times 17 \times 73$

$= 2 \times 9 \times 17 \times 73$

$= 18 \times 17 \times 73$

$= 306 \times 73 = 22338$

2	306	3	657
3	153	3	219
3	51		73
	17		

$\therefore$ the smallest number that is divisible by both 306and 657 is 22338

Section D

∗This section consists of 4 questions of 5 marks each

32. Determine graphically the coordinates of the vertices of triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

Solution: Given

$2y - x = 8;$
$x = 2y - 8$

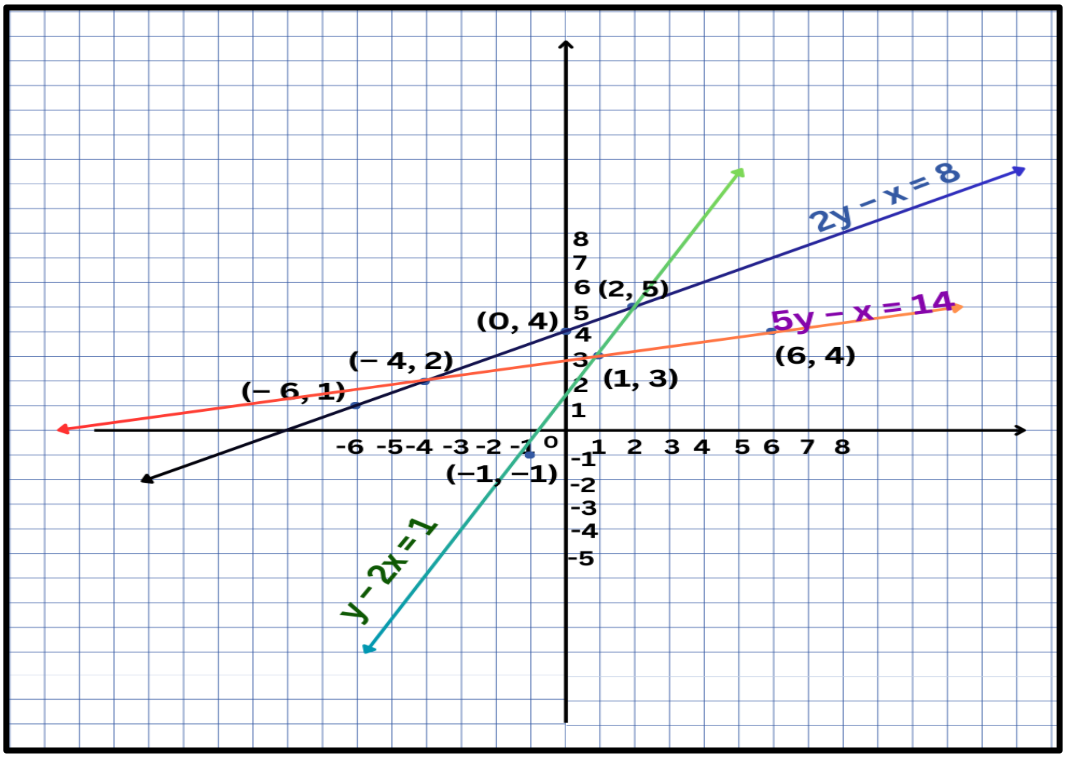
x:	− 6	− 4	0
y:	1	2	1
(x, y)	(− 6, 1)	(− 4, 2)	(0, 4)

$5y - x = 14$
$x = 5y - 14$

x:	−1	1	6
y:	−1	3	4
(x, y)	(−1, −1)	(1, 3)	(6, 4)

$y - 2x = 1$
$y = 1 + 2x$

x:	−1	1	2
y:	−1	3	5
(x, y)	(−1, −1)	(1, 3)	(2, 5)



From graph the coordinate of the vertices of a triangle are $(-4, 2)$, $(1,3)$, $(2,5)$

(OR)

Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the X-axis and shade the triangular region.

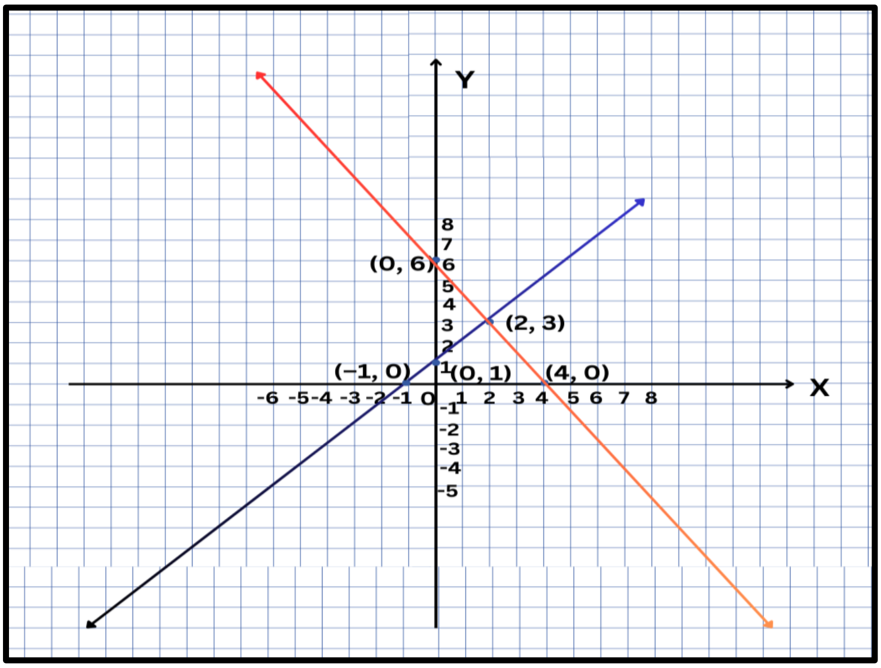
Solution: Given, Equations are $x - y + 1 = 0$ and $3x + 2y - 12 = 0$

$x - y + 1 = 0$

x:	0	− 1
y:	− 1	0
(x, y)	(0, − 1)	(− 1, 0)

$3x + 2y - 12=0$

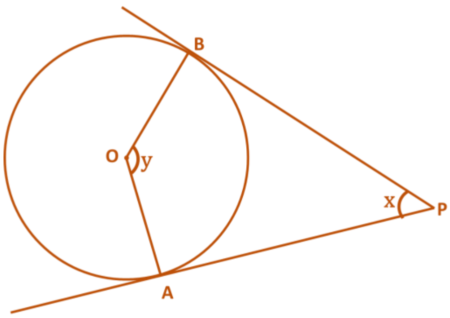
x:	0	4
y:	6	0
(x, y)	(0, 6)	(4, 0)



From figure, the vertices of triangle are $(-1,0),(2,3),(4,0)$

33. Two tangents PA and PB are drawn from an external point P to a circle with center O , such that $\angle APB = \angle x$ and $\angle AOB = \angle y$. Prove that opposite angles are supplementary or $\angle x + \angle y = 180$

Solution: Given two tangents PA and PB are drawn from an external point P to a circle with center O, such that $\angle APB = \angle x$ and $\angle AOB = \angle y$



$\angle OBP = 90^0$

$\angle OAP = 90^0$

In quadrilateral OAPB

$\angle OAP + \angle OBP + \angle AOB + \angle APB = 360^0$

$90^0 + 90^0 + \angle y + \angle x = 360^0$

$180^0 + \angle y + \angle x = 360^0$

$\angle y + \angle x = 360^0 - 180^0$

$\angle y + \angle x = 180^0$

opposite angles are supplementary or $\angle x + \angle y = 180$

34 The person standing on the bank of river observes that the angle of elevation of the top of a tree standing on opposite bank is 60^0. When he moves 30m away from the bank, he finds the angle of elevation to be 30^0. Find the height of tree and width of the river.

Solution:

Let AB=h be the height of the tree and
BD=x be the breadth of the river.
From the figure

$\angle ACB = 30^0$ and $\angle ADB = 60^0$

In Δ ABD, $\tan 60^0 = \frac{AB}{BD}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x$.............. (1)

In Δ ABC, $\tan 30^0 = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{40 + x}$

$\Rightarrow \sqrt{3}h = 40 + x$............... (2)

From (1) and (2)

$\sqrt{3}(\sqrt{3}x) = 40 \;\; x$

$\Rightarrow 3x = 40 + x$

$\Rightarrow 3x - x = 40$

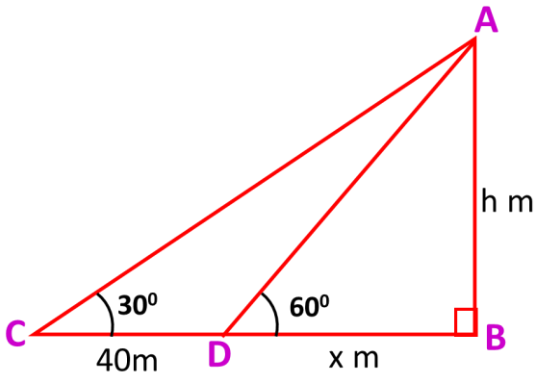
$\Rightarrow 2x = 40$

$\Rightarrow x = 20$

From (1)

$h = \sqrt{3}x = 20\sqrt{3} = 20 \times 1.732 = 34.64$m

$\therefore$ Height of the tree=34.64 m and width of the river=20m



(OR)

As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are30c and 45c. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships [Use 3 = 1.732]

Solution:

Let, height of light house from sea level (AB)=100m

Let, two ships be at the positions be C and D

In Δ ABD, $\tan 30^0 = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BD}$

$\Rightarrow BD = 100\sqrt{3}$

$= 100\,(1.732)$ [$\sqrt{3}$=1.732]

$=173.2$

In Δ ABC, $\tan 45^0 = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{AB}{BC}$

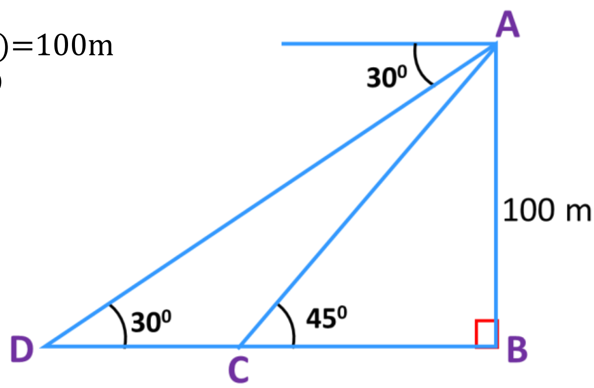
$\Rightarrow AB = BC = 100$

Now CD=BD − BC

$=173.2 - 100$

$=73.2$m

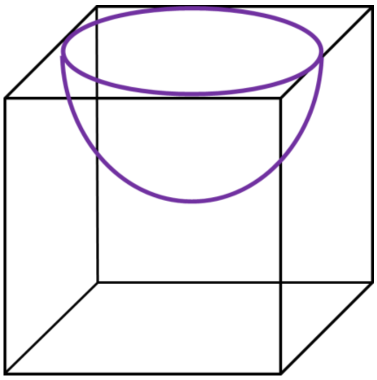
$\therefore$ The distance between two ship 73.2 m



35. A hemispherical depression is cut from one face of a cubical block, such that diameter l of hemisphere is equal to the edge of cube. Find the surface area of the remaining solid.

Solution:

Consider the diagram shown below.



It is given that a hemisphere of radius $\frac{l}{2}$ is cut out from the top face of the cuboidal wooden block.

Therefore, surface area of the remaining solid

$=$ surface area of the cuboidal box whose each edge is of length l $-$ Area of the top of the hemispherical part $+$ curved surface area of the hemispherical part

$= 6l^2 - \pi r^2 + 2\pi r^2$

$= 6l^2 + \pi r^2$

$= 6l^2 + \pi\left(\frac{l}{2}\right)^2$

$= 6l^2 + \pi \times \frac{l^2}{4}$

$= l^2\left(6 + \frac{1}{4}\right)$ square units

Section E

***This section consists of 3 questions of 4 marks each**

36. Case Study – 1

Maximum Profit: A kitchen utensils manufacturer can produce up to 200 utensils per day. The profit made from the sale of these utensils can be modeled by the function $P(x) = -0.5x + 175x - 330$, where P(x) is the profit in Rupees, and x is the number of utensils made and sold. Based on this model,

(i) Find the Y - intercept and explain what it means in this context.

(ii) Find the X - intercepts and explain what they mean in this context.

(iii) How many utensils should be sold to maximize profit?

(OR)

What is the maximum profit?

Solution:

(i) We get y-intercept by putting x=0 in $P(x) = -0.5x\ 2 + 175x - 3300$

$P(0) = -3300$

That means when no utensils made and sold, and then there is loss of Rs.3300.

(ii) We get x-intercept by putting $y = P(x) = 0$ in $P(x) = -0.5x^2 + 175x - 3300 = 0$

$$-0.5x^2 + 175x - 3300 = 0$$
$$0.5x^2 - 175x + 330 = 0$$
$$x^2 - 350x + 6600=0$$
$$x^2 - 330x - 20x + 6600=0$$
$$(x - 330)(x - 20)=0$$
$$x=330 \text{ or } x = 20$$

that means when the manufacturer produces 330 or 20 units of utensils, then its profit is 0.

(iii) As the profit function is quadratic in x, then maximum value of function occurs at $x=-\frac{b}{2a}$

For $P(x) = -0.5x^2 + 175x - 3300$

$$x= -\frac{b}{2a}$$
$$= -\frac{175}{2 \times 0.5} = 175$$

Hence 175 utensils should be sold to maximize profit.

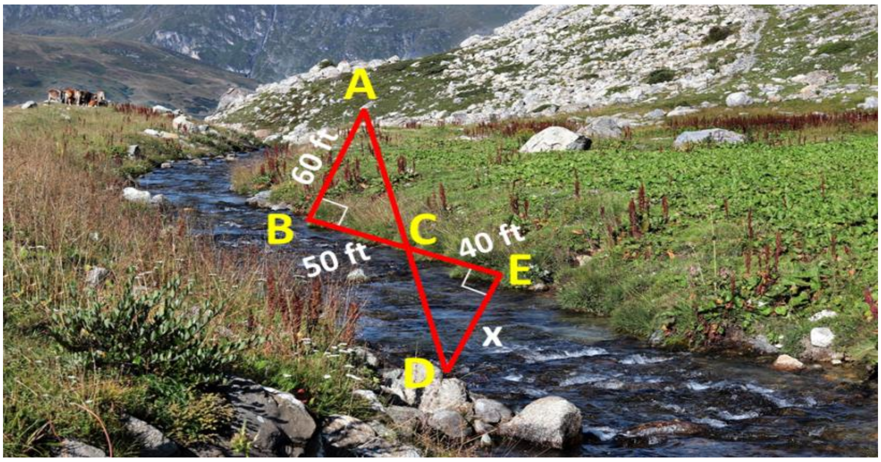
(OR)

Maximum profit = P (175)

$$= -0.5(175)^2 + 175(175) - 3300$$
$$= -0.5(30625) + 30,625 - 3300$$
$$= -15,312.5 + 30625 - 3300$$
$$= 30625 - 18,612.5$$
$$= 12,012.5$$

37. Case Study – 2

Tania is very intelligent in maths. She always try to relate the concept of maths in daily life. One day she plans to cross a river and want to know how far it is to the other side. She takes measurements on her side of the river and makes the drawing as shown below.



(i) Which similarity criterion is used in solving the above problem?

(ii) Consider the following statement:

$S_1 : \angle ACB = \angle DCE$ $S_2 : \angle BAC = \angle CDE$

Which of the above statement is/are correct?

(a) S_1 and S_2 both (b) S_1 (c) S_2 (d) None

(iii) Consider the following statement:

S_3: $\frac{AB}{DE} = \frac{CA}{DC}$

$S_4: \frac{BC}{CE} = \frac{AB}{DE}$

$S_5: \frac{CA}{DC} = \frac{DE}{AB}$ Which of the above statements are correct?

(a) S_3 and S_5 (b) S_4 and S_5 (c) S_3 and S_4 (d) All three

(iv) What is the distance x across the river?

(OR)

What is the approximate length of AD shown in the figure?

Solution:

(i) ΔABC and ΔDEC

$\angle ACB = \angle DCE$ (Vertically opposite angles)

$\angle ABC = \angle DEC = 90°$

$\Delta ABC \sim \Delta DEC$ AA similarity criterion

AA similarity criterion is correct answer

Angle-Angle Similarity (AA) Theorem: If two angles of one triangle and two angles of another triangle have the same measures, then the triangles are similar.

(ii) $\angle ACB = \angle DCE$

$\angle BAC \neq \angle CED$

Only S_1 is correct

(iii) $\frac{x}{60} = \frac{40}{50}$

$x = 48$

$\frac{AB}{DE} = \frac{BC}{CE} = \frac{CA}{DC}$

S_1 correct S_2 not correct S_3 not correct

S_1 correct

$AB = 80$, $BC = 60$

$AC^2 = AB^2 + BC^2$

$= 80^2 + 60^2$

$= 6400 + 3600$

$= 10000$

$AC = 100$ ft

(OR)

$AD = AC + CD = 100 + 63 = 163$ ft

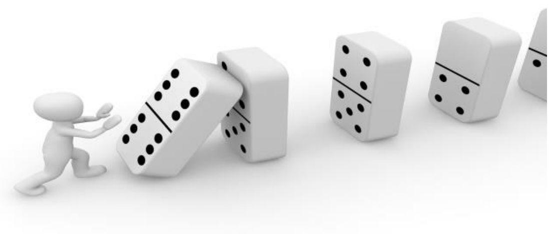
38. Case Study – 3

Double-six Dominos: It is a game played with the 28 numbered tiles shown in the diagram. The 28 dominos are placed in a bag, shuffled, and then one domino is randomly drawn. Give the following answer.

(i) What is the probability the total number of dots on the domino is three or less?

(ii) What is the probability the total number of dots on the domino is greater than three?

(iii) What is the probability the total number of dots on the domino does not have a blank half?



(OR)

What is the probability the total number of dots on the domino is not a "double" (both sides the same)?

Solution:

(i) Total no. of possible combinations - 28

No. of possible combinations with two ends having total number of dots on the domino is three or less - 6

Probability- No. of desired possible outcomes/Total no. of outcomes= $\frac{6}{28} = \frac{3}{14}$

(ii) Total no. of possible combinations - 28

No. of possible combinations with two ends having total number of dots on the Domino is greater than three - 22

Probability $= \frac{22}{28} = \frac{11}{14}$

(iii) Total no. of possible combinations - 28

No. of possible combinations with two ends having total number of dots on the domino, so that there is no blank half- 21

Probability - $\frac{21}{28} = \frac{3}{4}$

(OR)

Total no. of possible combinations – 28

Total number of dots on the domino is not a "double" (both sides the same) =23

The probability the total number of dots on the domino is not a "double" (both sides the same) $= \frac{23}{28}$

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