TS INTER MATHS -1A

Model Papers

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TS INTER

MATHS 1A

PRACTICE PAPER -1

By Satyam

MATHS - IA **TOTAL MARKS: 75**

MODEL PAPER - 1 **TIME: 3hrs.**

I. Very short answer type questions $10 \times 2 = 20$

1. If $A = \{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\}$ and $f : A \longrightarrow B$ is a surjection defined by $f(x) = \cos x$, find B.
2. Find the domain and range of the function $\frac{1}{\log(2-x)}$.
3. Find the inverse and adjoint of matrix $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$
4. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \\ 5 & -6 & x \end{bmatrix}$ and $\det A = 45$ then find x.
5. Let $\overline{OA} = \bar{i} + \bar{j} + \bar{k}$, $\overline{AB} = 3\bar{i} - 2\bar{j} + \bar{k}$, $\overline{BC} = \bar{i} + 2\bar{j} - 2\bar{k}$ and $\overline{CD} = 2\bar{i} + \bar{j} + 3\bar{k}$, then find the vector $\overline{OD}$.
6. Find the vector equation of the line joining the points $2\bar{i} + \bar{j} + 3\bar{k}$ and $-4\bar{i} + 3\bar{j} - \bar{k}$
7. If the vectors $\lambda\bar{i} - 3\bar{j} + 5\bar{k}$, $2\lambda\bar{i} - \lambda\bar{j} - \bar{k}$ are perpendicular to each other then find the value of λ.
8. Prove that $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = 4$.
9. Find cosine function whose period is 7.
10. If $\cos hx = \sec\theta$ then prove that $\tanh^2 \frac{x}{2} = \tan^2 \frac{\theta}{2}$

II. Short answer type questions $5 \times 4 = 20$

11. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ then show that $A^{-1} = A^T$.
12. Show that the line joining the pair of points $6\bar{a} - 4\bar{b} + 4\bar{c}$, $-4\bar{c}$ and the line joining the pair of points $-\bar{a} - 2\bar{b} - 3\bar{c}$ and $\bar{a} + 2\bar{b} - 5\bar{c}$ intersects at the point $-4\bar{c}$ when $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar vectors.
13. Prove that angle between two diagonals of a cube is given by $\cos\theta = \frac{1}{3}$.

14. Prove that $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=\frac{3}{2}$.

15. Solve: $1+\sin^2\theta = 3\sin\theta.\cos\theta$

16. Find the value of $\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$.

17. $\sin\theta=\frac{a}{b+c}$, then prove that $\cos\theta=\frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$.

III. **Long answer type questions** **$5\times 7 = 35$**

18. If $f: A \longrightarrow B$, $g: B \longrightarrow C$ are two bijective functions then prove that $gof : A \longrightarrow C$ is bijective function.

19. Using the principle of mathematical induction prove that $49^n + 16n - 1$ is divisible by $64\ \forall\ n \in Z^+$.

20. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2\,(a+b+c)^3$.

21. Solve the following system of equations by using matrix inversion method $x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$.

22. Let $\bar{a}, \bar{b}, \bar{c}$ be three vectors, then show that $(\bar{a}\times\bar{b})\times\bar{c} = (\bar{a}\cdot\bar{c})\,\bar{b} - (\bar{b}\cdot\bar{c})\,\bar{a}$ and $\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}\cdot\bar{c})\,\bar{b} - (\bar{a}\cdot\bar{b})\,\bar{c}$.

23. If $A + B + C = 2S$, then prove that

$$\cos(S-A)+\cos(S-B)+\cos(S-C)+\cos S = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}.$$

24. If $r_1 = 2$, $r_2 = 3$, $r_3 = 6$ and $r = 1$, then prove that $a = 3$, $b = 4$ and $c = 5$.

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TS INTER

MATHS 1A

PRACTICE PAPER - 2

By Satyam

MATHS - IA **TOTAL MARKS: 75**

MODEL PAPER - 2 **TIME: 3hrs.**

I. Very short answer type questions $10 \times 2 = 20$

1. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$ $\forall\, x \in R$ then find (i) (gof) (x) (ii) (fog) (x).

2. Find the domain of the real valued function $f(x) = \sqrt{4 - x^2}$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then find 3B – 2A.

4. If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew symmetric matrix, then find the value of x.

5. If $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$ and $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors then find the values of m and n.

6. Find the vector equation of the plane which passes through the points $2\bar{i} + 4\bar{j} + 2\bar{k}$, $2\bar{i} + 3\bar{j} + 5\bar{k}$ and parallel to the vector $3\bar{i} - 2\bar{j} + \bar{k}$.

7. Find the angle between the planes $r.(2\bar{i} - \bar{j} + 2\bar{k}) = 3$ and $r.(3\bar{i} + 6\bar{j} + \bar{k}) = 4$.

8. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

9. Find the period of the function $f(x) = \cos\left(\frac{4x+9}{5}\right)$.

10. If $\sinh x = \frac{3}{4}$ then find cosh 2x and sinh 2x.

II. Short answer type questions $5 \times 4 = 20$

11. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ then show that $(aI + bE)^3 = a^3 I + 3a^2 b$.

12. $\bar{a}, \bar{b}, \bar{c}$ are non – coplanar vectors. Prove that the following four points are coplanar $-\bar{a} + 4\bar{b} - 3\bar{c}$, $3\bar{a} + 2\bar{b} - 5\bar{c}$, $-3\bar{a} + 8\bar{b} - 5\bar{c}$ and $-3\bar{a} + 2\bar{b} + \bar{c}$

13. Find the unit vector perpendicular to the plane passing through the points (1, 2, 3), (2, – 1, 1) and (1, 2, –4).

14. Prove that $\left(1 + \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)\left(1 + \cos\frac{7\pi}{10}\right)\left(1 + \cos\frac{9\pi}{10}\right) = \frac{1}{16}$

15. If $0 < \theta < \pi$, solve $\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$.

16. Prove that $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{36}{85}$.

17. In Δ ABC, prove that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$.

III. **Long answer type questions** **$5 \times 7 = 35$**

18. If $f: A \longrightarrow B$, I_A and I_B are two identity functions on A and B respectively, then show that $foI_A = f = I_Bof$.

19. Using the principle of mathematical induction show that

$$\frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \cdots\cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1} \quad \forall\, n \in N.$$

20. If $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is non – singular matrix then show that A is invertible and $A = \frac{\text{adj } A}{\det A}$.

21. Solve the following system of equations by Gauss Jordan method

$x - y + 3z = 5$, $4x + 2y - z = 0$, $-x + 3y + z = 5$.

22. Find the shortest distance between the skew lines $(6\bar{i} + 2\bar{j} + 2\bar{k}) + t\,(\bar{i} - 2\bar{j} + 2\bar{k})$ and $(-4\bar{i} - \bar{k}) + s\,(3\bar{i} - 2\bar{j} - 2\bar{k})$ where s, t are scalars.

23. If $A + B + C = 180^0$, then prove that

$$\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2} = 4\cos\frac{\pi - A}{4}\cos\frac{\pi - B}{4}\cos\frac{\pi - C}{4}.$$

24. Show that $r + r_3 + r_1 - r_2 = 4r\cos B$.

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TS INTER

MATHS 1A

PRACTICE PAPER - 3

By Satyam

MATHS - IA **TOTAL MARKS: 75**

MODEL PAPER - 3 **TIME: 3hrs.**

I. Very short answer type questions $10 \times 2 = 20$

1. If f: R ⟶ R is defined by $f(x) = \frac{1 - x^2}{1 + x^2}$, then show that $f(\tan\theta) = \cos 2\theta$.
2. If f = {(1, 2), (2, – 3), (3, – 1)}, the find (i) 2f (ii) 2 + f (iii) f^2 (iv) $\sqrt{f}$.
3. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = 0$, then find the value of k.
4. Define trace of matrix and find the trace of A if $A = \begin{bmatrix} 1 & 2 & -\frac{1}{2} \\ 0 & -1 & 2 \\ -\frac{1}{2} & 2 & 1 \end{bmatrix}$
5. If $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}$ and $\bar{b} = \bar{i} + \bar{j} + \bar{k}$ and $\bar{c} = \bar{j} + 2\bar{k}$, then find the unit vector in the opposite direction of a + b + c.
6. Find the vector equation of the plane joining the points $\bar{i} - 2\bar{j} + 5\bar{k}$, $-5\bar{j} - \bar{k}$ and $-3\bar{i} + 5\bar{j}$.
7. If $4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$ is parallel to the vector $\bar{i} + 2\bar{j} + 3\bar{k}$, then find p.
8. Find the value of $\sin^2 82\frac{1}{2}° - \sin^2 22\frac{1}{2}°$.
9. Find the maximum and minimum values of the function $f(x) = 13\cos x + 3\sqrt{3}\sin x - 4$
10. Show that $\mathrm{Tanh}^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e 3$.

II. Short answer type questions $5 \times 4 = 20$

11. If $\theta - \emptyset = \frac{\pi}{2}$, then show that $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\emptyset & \cos\emptyset\sin\emptyset \\ \cos\emptyset\sin\emptyset & \cos^2\emptyset \end{bmatrix} = 0$
12. If ABCDEF is a regular hexagon with centre 'O', then show that $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{OA}$
13. If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}$, $\bar{b} = \bar{i} + \bar{j} - \bar{k}$ and $\bar{c} = \bar{i} - \bar{j} + \bar{k}$, then compute $\bar{a} \times (\bar{b} \times \bar{c})$ and verify that it is perpendicular to $\bar{a}$.

14. If A is not an integral multiple of $\frac{\pi}{2}$ then prove that cot A – tan A = 2 cot 2A.

15. Solve $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$.

16. Prove that $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$.

17. In Δ ABC, if $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, then show that $C = 60^0$.

III. **Long answer type questions** **5 × 7 = 35**

18. If f: Q ⟶ Q is defined by f(x) = 5x + 4 ∀ x ∈ Q, then show that f is bijective and find f^{-1}.

19. Using the principle of mathematical induction show that

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \cdots \text{ up to n terms } \frac{n}{24}[2n^2 + 9n + 13] \ \forall\, n \in N.$$

20. Show that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$.

21. Solve the following system of equations by Cramer's Rule

3x + 4y+ 5z =18, 2x – y + 8z = 13, 5x – 2y + 7z = 20.

22. If a line makes angles $\theta_1, \theta_2, \theta_3$ and θ_4 with the diagonals of a cube, then show that

$\cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 + \cos^2\theta_4 = \frac{4}{3}$

23. If A, B, C are angles in a triangle, then prove that

$\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$.

24. If $r : R ; r_1 = 2 : 5 : 12$, then prove that the triangle is right angle at A.

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TS INTER

MATHS 1A

PRACTICE PAPER - 4

By Satyam

MATHS - IA **TOTAL MARKS: 75**

MODEL PAPER - 4 **TIME: 3hrs.**

I. Very short answer type questions $10 \times 2 = 20$

1. If A = {– 2, – 1, 0, 1, 2} and f : A ⟶ B is a surjection defined by $f(x) = x^2 + x + 1$, then find B.
2. Find the domain of the real valued function $f(x) = \log(x^2 - 4x + 3)$.
3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and $2X + A = B$, then find X.
4. If $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$, then find the values of x, y, z and a.
5. If the vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}$ and $\mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear vectors, then find the values of μ and λ.
6. Find the vector equation of the line passing through the point $2\bar{i} + 3\bar{j} + \bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$.
7. If $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$, the show that $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ are perpendicular to each other.
8. If $\sin\theta = \frac{4}{5}$ and θ is not in the 1st quadrant, then find the value of cos θ.
9. Prove that $\sin 50^0 - \sin 70^0 + \sin 10^0 = 0$.
10. For any x ∈ R, then prove that $\cosh^4 x - \sinh^4 x = \cosh 2x$.

II. Short answer type questions $5 \times 4 = 20$

11. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then show that $A^{-1} = A^3$.
12. If the points $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $-\bar{i} + \bar{j} + 2\bar{k}$ and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$.
13. Find the volume of tetrahedron whose vertices are (1, 2, 1), (3, 2, 5), (2, – 1, 0) and (– 1, 0, 1).

14. Prove that $\cos\frac{\pi}{11}\cdot\cos\frac{2\pi}{11}\cdot\cos\frac{4\pi}{11}\cdot\cos\frac{5\pi}{11} = \frac{1}{32}$.

15. If θ_1, θ_2 are the solution of the equation $a\cos 2\theta + b\sin 2\theta = c$, $\tan\theta_1 \neq \tan\theta_2$ and $c + a \neq 0$, then find the values of (i) $\tan\theta_1 + \tan\theta_2$ (ii) $\tan\theta_1 \cdot \tan\theta_2$

16. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then prove that

$$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz.$$

17. If $a = (b + c)\cos\theta$, then prove that $\sin\theta = \frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$.

III. **Long answer type questions** **$5 \times 7 = 35$**

18. If $f: A \longrightarrow B$ is a bijection, then show that $fof^{-1} = I_B$ and $f^{-1}of = I_A$.

19. Using the principle of mathematical induction show that

$2\cdot3 + 3\cdot4 + 4\cdot5 + \ldots\ldots\ldots$ up to n terms$= \frac{n}{3}[n^2 + 6n + 11]\ \forall\, n \in N$.

20. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0$, then show that $abc = -1$.

21. Examine whether the following system of equations is consistent or inconsistent if it is consistent find the complete solution

$x + y + z = 3,\ 2x + 2y - z = 3,\ x + y - z = 1$.

22. If $\bar{a} = 3\bar{i} - \bar{j} + 2\bar{k}$, $\bar{b} = -\bar{i} + 3\bar{j} + 2\bar{k}$, $\bar{c} = 4\bar{i} + 5\bar{j} - 2\bar{k}$ and $\bar{d} = \bar{i} + 3\bar{j} + 5\bar{k}$ then compute (i) $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})$ (ii) $(\bar{a} \times \bar{b}) \cdot \bar{c} - (\bar{a} \times \bar{d}) \cdot \bar{b}$

23. If A, B, C are angles in a triangle, then prove that

$$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2} = 2\left(1 + \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right).$$

24. If $a = 13, b = 14, c = 15$,

then show that $R = 65/8$, $r = 4$, $r_1 = 21/2$, $r_2 = 12$ and $r_3 = 14$.

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TS INTER

MATHS 1A

PRACTICE PAPER - 5

By Satyam

MATHS - IA

MODEL PAPER - 5

TOTAL MARKS: 75

TIME: 3hrs.

I. Very short answer type questions $10 \times 2 = 20$

1. If $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x}$ $\forall$ $x \in (0, \infty)$, then find (gof) (x).
2. Find the range of the real valued function $f(x) = \frac{x^2 - 4}{x - 2}$.
3. Construct 3×2 matrix whose elements are defined by $a_{ij} = \frac{1}{2}|i - 3j|$.
4. For any square matrix A show that AA' is symmetric.
5. Is the triangle formed by the vectors $3\bar{i} + 5\bar{j} + 2\bar{k}$, $2\bar{i} - 3\bar{j} - 5\bar{k}$ and $-5\bar{i} - 2\bar{j} + 3\bar{k}$ equilateral?
6. If α, β, γ are the angles made by the vector $3\bar{i} - 6\bar{j} + 2\bar{k}$ with the positive direction of the coordinate axes, then find cos α, cos β and cos γ.
7. Find the area of the parallelogram having $2\bar{i} - 3\bar{j}$ and $3\bar{i} - \bar{k}$ as adjacent sides.
8. For what values of x tn the first quadrant $\frac{2\tan x}{1 - \tan^2 x}$ is positive.
9. Eliminate θ from $x = a\cos^3\theta$, $y = b\sin^3\theta$.
10. Prove that $\tan(x - y) = \frac{\tan x - \tan y}{1 - \tan x \tan y}$.

II. Short answer type questions $5 \times 4 = 20$

11. If $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}$, then find adj A and A^{-1}.
12. In two-dimensional plane prove by using vector method, the equation of the line whose intercepts on the axes are a and b is $\frac{x}{a} + \frac{y}{b} = 1$.
13. Find the area of the triangle whose vertices are A (1, 2, 3), B (2, 3, 1) and C (3, 1, 2).
14. Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$.
15. Find all the values of x in (− π, π) satisfying the equation $8^{1 + \cos x + \cos^2 x + \cdots} = 4^3$.

16. Find the value of $\tan\left[\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right]$.

17. If $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in AP, then prove that a, b, c are in AP.

III. **Long answer type questions** **$5 \times 7 = 35$**

18. If the function f is defined by

$$f(x) = \begin{cases} x + 2 ; & x > 1 \\ 2 ; & -1 \le x \le 1 \\ x - 1 ; & -3 < x < -1 \end{cases}$$

then find the values of f (2), f (0), f (– 1.5), f(2) + f (– 2) and f (– 5).

19. Using the principle of mathematical induction show that

$3.5^{2n+1} + 2^{3n+1}$ is divisible by 17, $\forall\, n \in N$.

20. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2ac - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$

21. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, then find $A^3 - 3A^2 - A - 3I$.

22. If $\bar{a}, \bar{b}, \bar{c}$ are non-zero vectors and $\bar{a}$ is perpendicular to both $\bar{b}$ and $\bar{c}$ if $|\bar{a}| = 2$, $|\bar{b}| = 3$

$|\bar{c}| = 4$ and $(\bar{b}, \bar{c}) = \frac{2\pi}{3}$, then find $|[\bar{a}\ \bar{b}\ \bar{c}]|$.

23. If A, B, C are angles in a triangle, then prove that

$\operatorname{Cos} A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

24. Prove that $\dfrac{\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}}{\cot A + \cot B + \cot C} = \dfrac{(a + b + c)^2}{a^2 + b^2 + c^2}$