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TS

X CLASS

Mathematics

SOLVED QUESTION PAPER-1

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SOLVED PRACTICE PAPER - 1

SUBJECT: MATHEMATICS **TIME: 3HRS.**

CLASS: X **TOTAL MARKS: 80**

PART – A

SECTION – I $(6 \times 2 = 12)$

1. Explain why $5 \times 11 \times 12 + 12$ is a composite number.

Solution:

Note: Every composite number can be expressed as a product of primes.

Given numbers is $5 \times 11 \times 12 + 12$

$= 12(5 \times 11 + 1)$

$= 12(55 + 1) = 12\ (56)$

$= 12 \times 2 \times 28$

$= 4 \times 3 \times 2 \times 4 \times 7$

$= 2^2 \times 3 \times 2 \times 2^2 \times 7$

$= 2^5 \times 3 \times 7$

= Product of prime factors Hence the given number is a composite number.

$\therefore 5 \times 11 \times 12 + 12$ is a composite number

2. Write the roster form of the set $A = \{x : x = 2n + 1\ \forall\ n \in N\}$

Solution:

If $n = 1$ then $2n + 1 = 2(1) + 1$
$= 2 + 1 = 3$

If $n = 2$ then $2n + 1 = 2(2) + 1$
$= 4 + 1 = 5$

If $n = 3$ then $2n + 1 = 2(3) + 1$
$= 6 + 1 = 7$

So {3, 5, 7, 9.......} is the roster form of given set.

3. Find p(3) if $p(x) = x^2 – 5x – 6$ is given.

Solution:

$p(x) = x^2 – 5x + 6$ (given)

$p(3) = 3^3 – 5(3) + 6$

$= 9 – 15 + 6 = 15 – 15 = 0$

$\therefore p(3) = 0.$

4. For what value of k, the following system of equations has a unique solution, $x – ky = 2;\ 3x + 2y = – 5$

Solution:

$a_1 x + b_1 y + c_1 = 0$ and

$a_2 x + b_2 y + c_2 = 0 = 0$ will have a unique solution.

will have a unique solution.

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ in the given system of equations

$a_1 = 1, b_1 = – k, a_2 = 3, b_2 = 2$

$\frac{1}{3} \neq \frac{-k}{2}$

So the system of equations will have a unique solutions for $k = R – (-\frac{2}{3})$

5. A ladder 13m long reaches a window of building 12cm above the ground. Determine the distance of the foot of the ladder from the building.

Solution:

In Δ ABC, B = $\angle 90°$

$\Rightarrow AC^2 = AB2^2 + BC^2$ (By Pythagoras theorem)

Let AC = 13m, AB = 12m, BC = ?

$\Rightarrow 13^2 = 12^2 + BC^2$

$\Rightarrow 169 = 144 + BC^2$

$\Rightarrow BC^2 = 169 - 144 = 25$

$\Rightarrow BC = \sqrt{25} = 5m$

Hence, the foot of the ladder from the building is at a distance of 5m.

6. A boat has to cross a river. It crosses river by making an angle of 60° with bank, due to the stream of river it travels a distance of 450 m to reach another side of river. Draw a diagram to this data

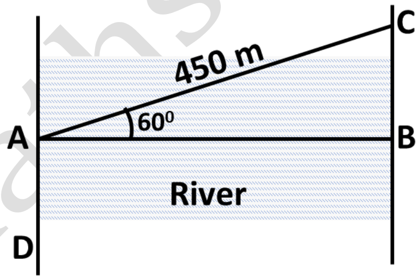
Solution:

AB is the width of river

AD, BC are river banks

AC is the distance travelled by boat in river = 450 m

A is the initial point, C is the terminal point



SECTION – II (6 × 4 = 24)

7. Prove that $\sqrt{2} + \sqrt{5}$ irrational.

Solution:

Let us assume to the contrary that $\sqrt{2} + \sqrt{5}$ is a rational number.

Then, there exist co-prime positive integers 'a' and 'b' such that

$\sqrt{2} + \sqrt{5} = \frac{a}{b}$

$\sqrt{2} = \frac{a}{b} - \sqrt{5}$

Squaring on both sides

$\left(\sqrt{2}\right)^2 = \left(\frac{a}{b} - \sqrt{5}\right)^2$

$2 = \frac{a^2}{b^2} - 2\sqrt{5}\,\frac{a}{b} + \left(\sqrt{5}\right)^2$

$2\sqrt{5}\,\frac{a}{b} = \frac{a^2}{b^2} + 5 - 2$

$2\sqrt{5}\,\frac{a}{b} = \frac{a^2}{b^2} + 3$

$2\sqrt{5}\,\frac{a}{b} = \frac{a^2 + 3b^2}{b^2}$

$\sqrt{5} = \frac{a^2 + 3b^2}{2ab}$

Since a, b are integers $\frac{a^2 + 3b^2}{2ab}$ is rational, and so $\sqrt{5}$ is also rational

But this is a contradiction to the fact that $\sqrt{5}$ is an irrational number.

So our assumption is wrong. $\therefore \sqrt{2} + \sqrt{5}$ is irrational.

8. Laxmi does not want to disclose the l, b, h of a cuboid of her project. She has constructed a polynomial $x^3 - 6x^2 + 11x - 6$ by taking the values of l, b, h as its zeros. Can you open the secret?

Solution:

$P(x) = x^3 - 6x^2 + 11x - 6$

$P(1) = (1)^3 - 6(1)^2 + 11(1) - 6$

$= 1 - 6 + 11 - 6$

$= 12 - 12 = 0$

$\therefore$ for $P(x)$, $(x - 1)$ is a factor

$x^2 - 5x + 6 = x^2 - 3x - 2x + 6$

$= x(x - 3) - 2(x - 3)$

$= (x - 3)(x - 2)$

$P(x) = (x - 1)(x - 2)(x - 3)$

$P(x)$ zeroes of the polynomial are 1, 2, 3

$\therefore$ Measurements of the cuboid are 1, 2 and 3 units.

Its solution doesn't change. So it is true.

9. Solve the following pair of linear equations by substitution method,

$2x - 3y = 19$ and $3x - 2y = 21$

Solution:

The given equations are

$2x - 3y = 19$ —– (1) and $3x - 2y = 21$ —– (2)

From the equation (1)

$2x = 19 + 3y \Rightarrow x = \frac{19 + 3y}{2}$

Now substituting this value of $x = \frac{19 + 3y}{2}$ in equation (2) we get

$3x - 2y = 21$ becomes

$3\left(\frac{19 + 3y}{2}\right) - 2(y) = 21$

$\Rightarrow 57x + 9y - 4y = 21 \times 2$

$9y - 4y = 42 - 57$

$5y = -15$

$\therefore y = -155 = -3$

Now put $y = -3$ value in $x = \frac{19 + 3y}{2}$ we get

$x = \frac{19 + 3(-3)}{2}$

$x = \frac{19 - 9}{2}$

$x = \frac{10}{2}$

So $x = 5$ and $y = -3$ are the solutions of given equations.

Verification:

$2x - 3y = 19$

$2(5) - 3(-3) = 19$

$10 + 9 = 19$

$19 = 19$

LHS= RHS

$3x - 2y = 21$

$3(5) - 2(-3) = 21$

$15 + 6 = 21$

$21 = 21$

LHS = RHS

10. If $4\sin^2\theta - 1 = 0$ then find 'θ' ($\theta < 90$) also, find the value of θ and the value of $\cos^2\theta + \tan^2\theta$

Solution:

Given, $4\sin^2\theta - 1 = 0$

$4\,\text{Sin}^2\,\theta = 1$

$\text{Sin}^2\,\theta = \frac{1}{4}$

$\text{Sin}\,\theta = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

Given θ is less than 90°

$\therefore \text{Sin}\,\theta = \frac{1}{2}0$

$\sin\theta = \sin 30°$

$\therefore \theta = 30°$

$\text{Cos}\,\theta = \text{Cos}\,30° = \frac{\sqrt{3}}{2}$

$\text{Tan}\,\theta = \text{Tan}\,30° = \frac{1}{\sqrt{3}}$

$\cos^2\theta + \tan^2\theta = \cos^2 30° + \tan^2 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2$

$= \frac{3}{4} + \frac{1}{3} = \frac{9+4}{12} = \frac{13}{12}$

11. Find the trisection points of the line segment joined by the points (–3, 3) and (3, –3).

Solution:

The points which divide the line segment by 1:2 and 2:1 ratio (internally) is called trisection points.

Formula for the points of trisection of the line segment joined by (x_1, y_1) and (x_2, y_2) are

$= \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Where m = 2 and n = 1.

Here $x_1 = -3$, $y_1 = 3$ and $x_2 = 3$, $y_2 = -3$, then the point in the ratio 2 : 1 is

So (1, –1) is one trisection point.

The point which is at 1 : 2 ratio is another trisection point.

So m = 1, n = 2.

Here $x_1 = -3$, $y_1 = 3$ and $x_2 = 3$, $y_2 = -3$

then (–1, 1) is another trisection point.

P, Q are trisection points.

12. 5 red and 8 white balls are present in a bag. If a ball is taken randomly from the bag then find the probability of it to be

i) White ball

ii) Not to be white ball

Solution:

Total number of balls present in bag

$= 5$ (red) $+ 8$ (white) $= 13$

Number of white balls $= 8$

Probability for taking out a white ball

P(taken white ball) $= \frac{8}{13}$

Probability for not to be a white ball $=$ 1 – P(taken white ball)

$= 1 - \frac{8}{13}$

$= \frac{5}{13}$

SECTION – III ($4 \times 6 = 24$)

13. If A = (x : Y is a Natural number below 10}

B = {x : Y is an even number below 10}

C = {x : Y is an odd number below 10}then

find (i) A – B (ii) A – C (iii) B ∪ C (iv) Also mention the Mutually disjoint sets among (i), (ii) and (iii).

Solution:

Given A = {1, 2, 3, 4, 5, 6, 7, 8, 9}, B = {2, 4, 6, 8} and C = {1, 3, 5, 7, 9}

i) A – B = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {2, 4, 6, 8}

= {1, 3, 5, 7, 9}

ii) A – C = {1, 2, 3, 4, 5, 6, 7, 8, 9} – {1, 3, 5, 7, 9}

= {2, 4, 6, 8}

iii) B ∪ C = {1, 2, 3, 4, 5, 6, 7, 8, 9}

iv) (i) and (ii) are disjoint sets because there is no element in common.

(ii) and (iii) are not disjoint sets because there are 2, 4, 6, 8 common elements.

(i) and (iii) are not disjoint sets because there are 1, 3, 5, 7, 9 common elements.

14. In a flower garden there are 23 plants in first row, 21 plants in second row, 19 plants in 3rd row and so on. If there are 10 rows in that garden, then find the to-tal number of plants in the last row with the help of the formula tn = a + (n – 1) d. (T.S. Mar. 16)

Solution:

No. of plants in 1st row = 23

No. of plants in 2nd row = 21

No. of plants in 3rd row = 19 and so on.

So the progression is 23, 21, 19, ...

in this A.P a = 23, d = 21 – 23 = – 2

n = 10

$a_n = a + (n - 1)$

$a_{10} = 23 + (10 - 1)(-2)$

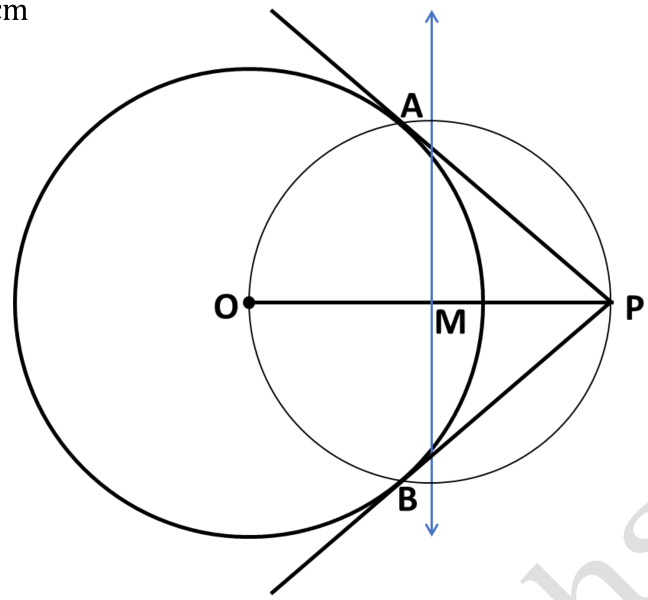
$= 23 + 9(-2)$

$= 23 - 18 = 5$

∴ Number of plants in the last row = 5.

15. Construct and measure the length of a pair of tangents that are drawn from a point at a distance of 8 cm whose radius is 5 cm

Solution:



Steps of construction:

Step1: Construct a circle with a radius of 5 cm.

Step2: Take the point 'P' in the exterior of the circle which is at a distance of '8' cm from its centre.

Step 3: Construct a perpendicular bisector to OP which meets at M.

Step 4: The draw a circle with a radius of MP or MO from the point M. This circle cuts the previous circle drawn from the centre 'O' at the points A and B.

Step 5: Now join the points PA and then PB.

PA, PB are the required tangents which are measured 6.2 cm long.

$OA = 5$ cm; $OP = 8$ cm

$AP = PB = 6.2$ cm.

16. A cone of height 16 cm and radius of base 4 cm is made up modelling clay. A child reshapes it into a sphere find the radius of the sphere.

Solution:

Given height of the cone = 16 cm

Radius = 4 cm

Volume of cone $= \frac{1}{3}\pi r^2\ h$

$= \frac{1}{3}\pi \times (4)^2 \times 16$

$= \frac{1}{3}\pi \times 4 \times 4 \times 16$

Since, the volume of the clay is in the form of the cone and the sphere remains the same

Volume of sphere = Volume of the cone

$\frac{4}{3}\pi r^3 = 13\ \pi \times 4 \times 4 \times 16$

$r^3 = 13 \times 4 \times 4 \times 16 \times 34$

$r^3 = 64 = 4^3$

$\therefore r = 4$

$\therefore$ The radius of the sphere = 4

17. Find the mode of the following data

Monthly Consumption	60 – 80	80 – 100	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
No. of Consumers	8	10	16	20	14	6	5

Solution:

Since, the maximum number of consumers (is 20) have got monthly consumption in the interval 120 – 140, the modal class is 120 – 140.

The lower boundary (l) of the modal class = 120.

The class size (h) = 20

The frequency of modal class (f_1) = 20

The frequency of the class preceding the modal class (f_0) = 16.

The frequency of the class succeeding the modal class (f_2) = 14.

Now, using the formula:

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 120 + \frac{20 - 16}{2(20) - 16 - 14} \times 20$$

$$= 120 + \frac{4}{40 - 30} \times 20$$

$$= 120 + \frac{4}{10} \times 20$$

$$= 120 + 8 = 128$$

18. The Coach of a cricket team buys 3 bats and 6 balls for ₹ 3,900. Later he buys another bat and two more balls of the same kind for ₹ 1,300. What is the cost price of each? Solve the situation Graphically.

Solution:

Let the cost of a bat = ₹ x

The cost of a bat = ₹ y

It is given that, the cost of 3 hats and 6 balls together is ₹ 3,900

$\Rightarrow 3x + 6y = 3{,}900$

$\therefore x + 2y = 1{,}300$ —– (1)

It is also given that the cost of a bat and 2 balls together is ₹ 1,300.

$\therefore x + 2y = 1{,}300$ —- (2)

By the observations equation (1). equation (2) are dependent equation,

$x + 2y = 1{,}300$

$2y = 1{,}300 - x$

$y = 1{,}300 - x$

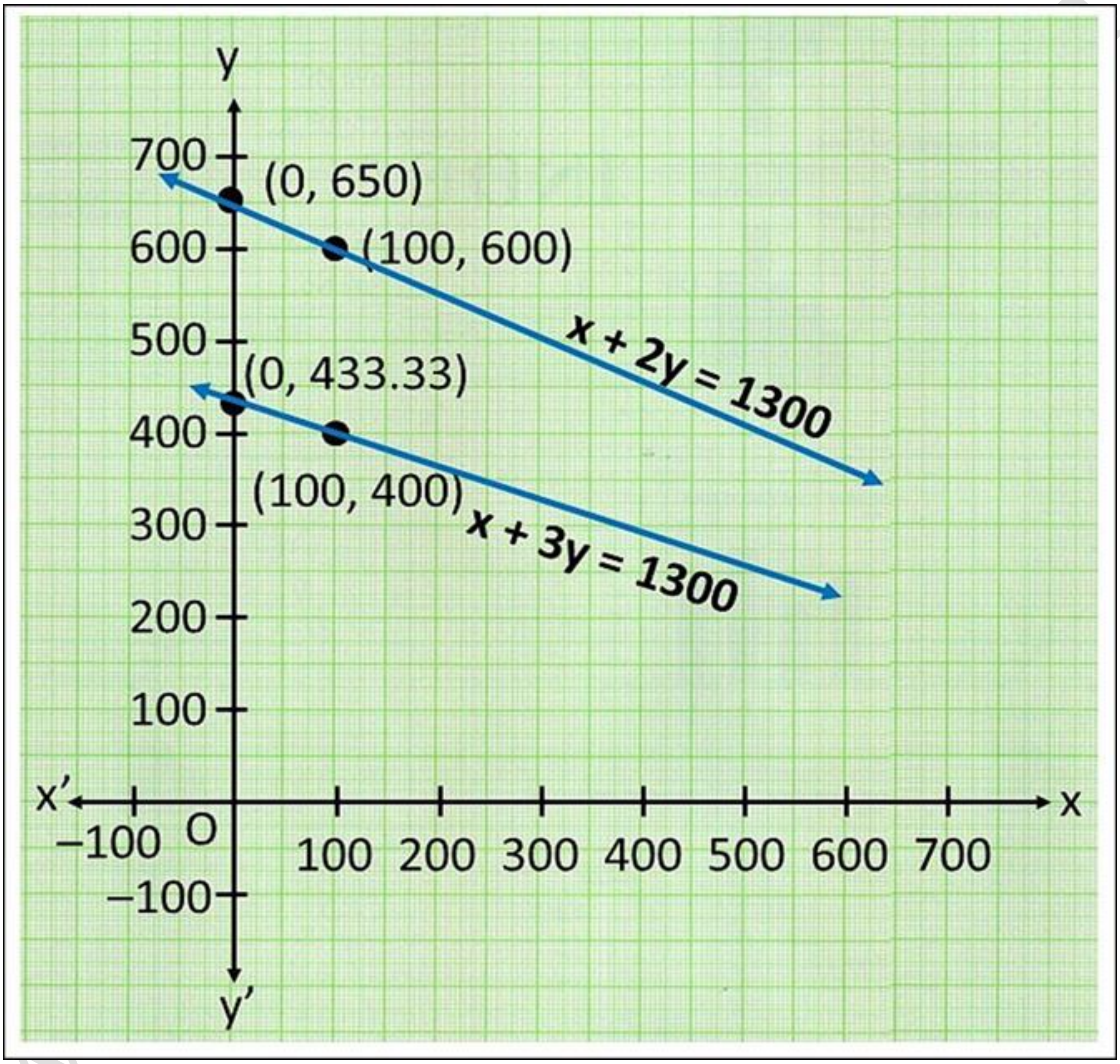
$y = \frac{1300 - x}{2}$

$x + 2y = 1300$

x	0	100
y	650	600

$x + 3y = 1300$

x	0	100
y	433.33	400



Given equations have infinitely many solutions

PART – B

Chose the correct answer **20 × 1=20**

1. The number of prime factors of 36 is (c)
   a) 4 b) 3 c) 2 d) 1
2. If $A \subset B$, then $A - B =$ (d)
   a) A b) B c) 0 d) ∅
3. The quadratic polynomial having $\frac{1}{3}$ and $\frac{1}{2}$ as its zeroes, is (c)
   a) $x^2 + \frac{5x+1}{6}$, b) $x^2 - \frac{5x+1}{6}$ c) $x^2 - \frac{5x-1}{6}$ d) $x^2 + \frac{5x+1}{6}$
4. If $2x + 3y = 8$ and $4x + py = 16$ has infinite solutions, then the value of p is (b)
   a) 8 b) 6 c) 10 d) 16
5. The sum of the roots of $6x^2 = 1$ is (a)
   a) 0 b) $\frac{1}{6}$ c) $-\frac{1}{6}$ d) 6
6. If a number is 132 smaller than its square, then the number is (d)
   a) 11 b) 8 c) 9 d) 12
7. If a, b, c are in GP, then $b^2 =$ (a)
   a) ac b) $\sqrt{ac}$ c) $(ac)^2$ d) $\frac{a+c}{2}$
8. If (x, y), (2, 0), (3, 2) and (1, 2) are vertices of a parallelogram, then (x, y) = (a)
   a) (0. 0) b) (4, 8) c) (1, 0) d) (5, 0)
9. The distance of (3, 4) from origin is (c)
   a) 1 b) 2 c) 5 d) 3
10. In ΔABC , DE ∥ BC, AD = 1.5 cm, DB = 6cm, AE = x cm, EC = 8cm, then x = ... (b)
    a) 2.5cm b) 2 cm c) 3cm d) 3.5 cm
11. The ratio of the corresponding sides of two similar triangles is 5 : 3, then the ratio of their areas is (d)
    a) 5:3 b) 3:5 c) 6:10 d) 25:9
12. Two sides of a right triangle are 3cm and 4cm, then the third side is (c)
    a) 9 b) 6 c) 5 d) 7
13. Number of secants that can be drawn to a circle through a point inside the circle is (a)
    a) 0 b) 1 c) 2 d) infinite
14. A Cylinder and a Cone have equal radii and equal heights. If the volume of Cylinder is 27cu., then the volume of Cone is (b)
    a) 27cu. b) 9 cu. c) 81 cu. d) 3 cu.
15. Which one of the following is not defined (c)
    a) Sin 90^0 b) Cos 0^0 c) Sec 90^0 d) Cos 90^0
16. $2 \sin\theta = \sin^2\theta$ is true for the values of θ is (a)
    a) 0^0 b) 45^0 c) 30^0 d) 60^0
17. A 20m long ladder is placed on a pole of 10m height making 'α' angle with the ground, then α is(d)
    a) 30^0 b) 60^0 c) 90^0 d) 45^0
18. If P(A) = 0.82, then $P(\overline{E}) =$ (a)
    a) 0.18 b) 0.28 c) 0.38 d) 0.48
19. On random selection, the probability of getting a composite number among the numbers from 51 to 100 (b)
    a) $\frac{1}{5}$ b) $\frac{4}{5}$ c) $\frac{4}{3}$ d) $\frac{7}{10}$
20. The mid value of the class 50 – 60 is (d)
    a) 50 b) 60 c) 45 d) 55