AS TUTORIAL IN TELUGU

TELANGANA STATE

10TH CLASS

Maths

CONCEPT & FORMULAE

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1. REAL NUMBERS

Rational number:

A number, which is written in the form of $\frac{p}{q}$ where p, q are integers and $q \neq 0$ is called rational number. It is denoted by Q.

Irrational numbers:

A number, which is not rational, is called irrational number. It is denoted by Q' or S.

Euclid division lemma:

For any two positive integers a and b, there exist two integers q, r uniquely satisfying the rule $a = bq + r$, $0 \leq r < b$

Prime number:

A number which has only two factors 1 and itself is called prime number.
2, 3, 5, 7 .... Etc. are prime numbers

Composite number:

A number that has more than two factors is called composite number.
4, 6, 8, 9, 10,... etc.

Co-prime numbers:

Two numbers are said to be co-prime numbers, if they have no common factor except 1
Ex: (1, 2) , (3, 4), (4, 7)...etc.

Note: HCF of co - prime numbers is always 1
LCM of co - prime numbers is always their product
Ex: HCF (2, 5) = 1
LCM $(2, 5) = 2 \times 5 = 10$

Fundamental Theorem of Arithmetic:

Every Composite number (positive) can be expressed as product of primes in a unique manner irrespective of their order.
Example: $24 = 2\times 2 \times 2 \times 3 = 2^3 \times 3$

To find HCF, LCM by using prime factorisation method:

H.C.F = product of the smallest power of each common prime factors of given numbers.

L.C.M = product of the greatest power of each prime factor of given numbers.

- Decimal numbers with the finite no. of digits is called terminating decimal.
- Decimal numbers with the infinite no. of digits is called non- terminating decimal.
- In a decimal, a digit or a sequence of digits in the decimal part keeps repeating itself infinitely. Such decimals are called non- terminating repeating decimals.
- In $\frac{p}{q}$, if prime factorisation of q is in the form $2^m\,5^n$, then $\frac{p}{q}$ is terminating decimal.

  Otherwise non-terminating repeating decimal
- 'p' is a prime number and 'a' is a positive integer, if p divides a^2, then p divides a.

Relationship between L.C.M and H.C.F of two numbers:

For any two positive integers 'a' and 'b'

H.C.F (a, b) × L.C.M (a, b) = a × b

Ex: Let the numbers be 4 and 12

H.C.F (4, 12) = 4 and L.C.M (4, 12) = 12

H.C.F (4, 12) × L.C.M (4, 12) = 48

a × b = 4 × 12 = 48

Logarithms:

If $a^x = N$ then $x = \log_a N$

Standard formulae of logarithms:

- $\log_a xy = \log_a x + \log_a y$
- $\log_a \frac{x}{y} = \log_a x - \log_a y$
- $\log_a x^m = m \log_a x$
- $\text{Log}_a 1 = 0$
- $\log_a a = 1$
- $a^{\log_a x} = x$
- $\log_a b = \frac{1}{\log_b a}$

.

2. SETS

"Set theory was developed by George canter"

Set: A collection of well-defined objects is called a set.

- Sets are usually denoted by English alphabet
- Elements in a set are written in a curly bracket { } separated by commas
- In a 'Roster form', we are writing a set by listing the elements in it.
  EX: $A = \{1, 2, 3, 4\}$; $B = \{a, e, i, o, u\}$
- In a 'Set builder form' we are write a set by defining its elements with a "Common property"
- Set builder form should follow some syntax

Ex: A ={x/ x is a natural number and x≤5}

the set of all x → {x ; such that → / ; Property assigned to x → x is a natural number and x≤5

$B = \{x$: x is an English alphabet$\}$

/ or : is read as "such that"

- The symbol '∈' is used to denote membership of an element and read as 'belongs to'

  Ex: $A = \{1, 2, 3, 4\}$

  $1 \in A, 4 \in A$ and $5 \notin A$

∈ = belongs to and ∉ = not belongs to

Empty set (Null set or Void set):

A set which has no elements is called as null set. It is denoted by ∅ or { }.

Ex: $A = \{x$: $1 < x < 2$, x is a natural number

Finite set:

A set that contains finite number of elements is called finite set

Ex: $A = \{4, 2, 3, 5\}$; $B = \{x: x \in w, 0 \le x \le 6\}$

Infinite set:

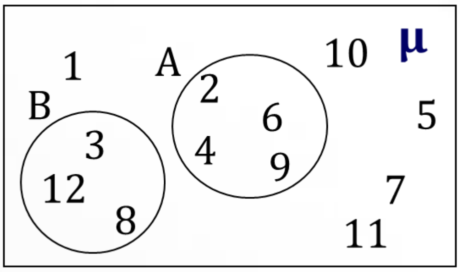
A set that contains infinite number of elements is called infinite set

Ex: $A = \{1, 2, 3, 4, \ldots\ldots\ldots\ldots\}$; $B = \{x : x > 5, x \in Z\}$

Venn Diagrams:

It is one of the ways of representing the relationships between sets. Venn diagrams are also called as "Venn – Euler diagrams"

- These diagrams consist of rectangles and closed curves like circles

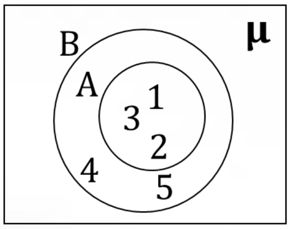


$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{2, 4, 6, 9\}$; $B = \{3, 8, 12\}$

Sub set:

For any two sets A and B, if every element of set A is in set B, then we can say that A is subset of B. It is denoted by $A \subset B$.

Ex: If $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$, then we say that 'A is subset of B', and symbolically $A \subset B$.



Note: (i) If a set has n elements, then no. of subsets of that set $= 2^n$

(ii) Empty set is subset of every set

(iii) Every set is a sub set of itself

If $A \subset B$, then $A \cup B = B$ and $A \cap B = A$, $A - B = \emptyset$

Power set:

Set of all the subsets of a set A are called power set of A. it is denoted by P(A).

Ex: $A = \{1, 2, 3\}$

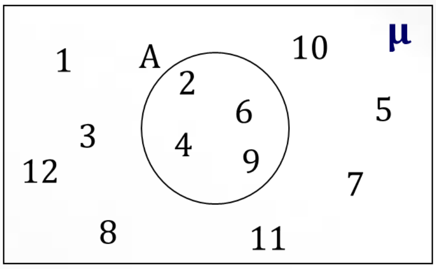
Subsets of A are: $\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}, \emptyset$

$P(A) = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}, \emptyset\}$

- If a set A has n elements, then no. of elements of $P(A) = 2^n$

Universal set:

A set which contains all the subset of it under our consideration is called universal set. The universal set is denoted by 'μ' or 'U' and represented by rectangles

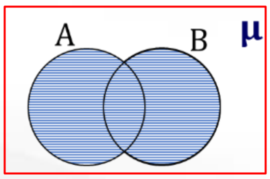


$\mu = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
$A = \{2, 4, 6, 9\}$

Union of sets:

The union of two sets A and B is the set which contains all the elements of set A or set B or both the sets A and B.

- The symbol '∪' is used to denote the union
- Union of A and B is written as $A \cup B$ and read as 'A union B'
- $A \cup B = \{x: x \in A \text{ or } x \in B\}$



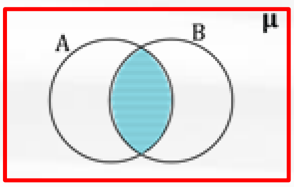
Ex: $A = \{1, 2, 3\}$, $B = \{1, 2, 4, 6\}$

$A \cup B = \{1, 2, 3\} \cup \{1, 2, 4, 6\} = \{1, 2, 3, 4, 6\}$

Intersection of sets:

The intersection of two sets A and B is the set which contains all the elements which are common in both the sets A and B.

- The symbol '∩' is used to denote the union
- Intersection of A and B is written as $A \cap B$ and read as 'A intersection B'
- $A \cap B = \{x: x \in A \text{ and } x \in B\}$



Ex: $A = \{1, 2, 3\}$, $B = \{1, 2, 4, 6\}$

$A \cap B = \{1, 2, 3\} \cap \{\{1, 2, 4, 6\} = \{1, 2\}$

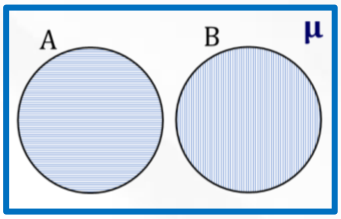
Disjoint sets:

Two sets A and B are said to be disjoint sets, if they have no common element

$\Rightarrow A \cap B = \emptyset$

Ex: $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$

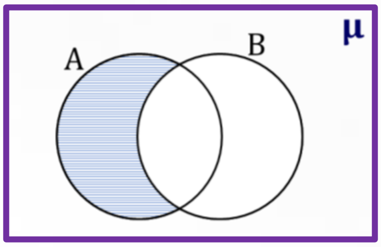
$A \cap B = \emptyset$



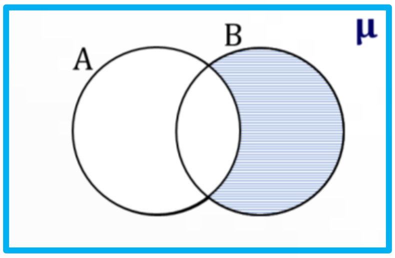
Difference of sets:

The difference set of sets A and B is the set of elements which belongs to A but not belongs to B

Difference of A and B is Denoted by A – B



A – B



B – A

$A - B = \{x: x \in A \text{ and } x \notin B\}$, $B - A = \{x: x \in B \text{ and } x \notin A\}$

Cardinal number of a set:

Number of elements in a set A is called cardinal number of that set A.

It is denoted by n (A).

Ex: If $A = \{a, e, I, o, u\}$, then $n(A) = 5$

Equal sets:

Two sets A and B are said to be equal sets, if they have same elements.

Equal sets denoted by $A = B$

Ex: $A = \{1, 3, 2\}$; $B = \{3, 2, 1\}$

$\therefore A = B$

Equivalent sets:

Two sets A and B are said to be equivalent sets if, $n(A) = n(B)$.

Equal sets denoted by $A \sim B$ or $A \equiv B$

Ex: $A = \{1, 3, 2\}$; $B = \{a, b, c\}$

$n(A) = 3$; $n(B) = 3$

$\therefore A \sim B$

3. POLYNOMIALS

Polynomial:
An algebraic expression becomes a polynomial if the powers of variable(s) are whole numbers.

Ex: $2x - 5, 3x^2 + 5x - 3$, $x^3 + 3x^2 - 6x + 3$

Value of a polynomial
p(a) is the value of a polynomial p(x) at x = a

Ex: let $p(x) = 2x - 5$

Put $x = 1 \Rightarrow p(1) = 2(1) - 5 = 2 - 5 = -3$

$\therefore -3$ is the value of the polynomial p(x) at x = 1

Zero of a polynomial
Zeroes of a polynomial p(x) is any real number 'k' such that p(k) = 0.

Ex: let $p(x) = x - 5$

Put $x = 5 \Rightarrow p(5) = 5 - 5 = 0$

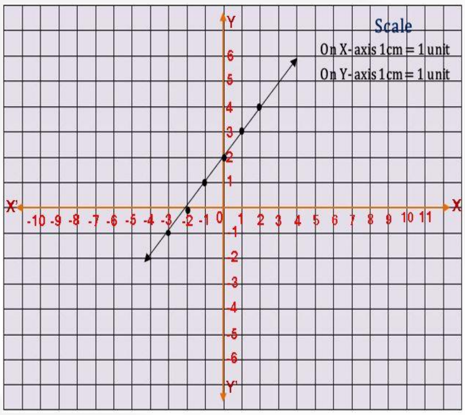
$\therefore$ 5 is the zero of the polynomial p(x)

Note: For finding the zeroes of the polynomial p(x), let p(x) = 0

Degree of a polynomial
The highest power of x in a polynomial p(x) is called as degree of the polynomial p(x)

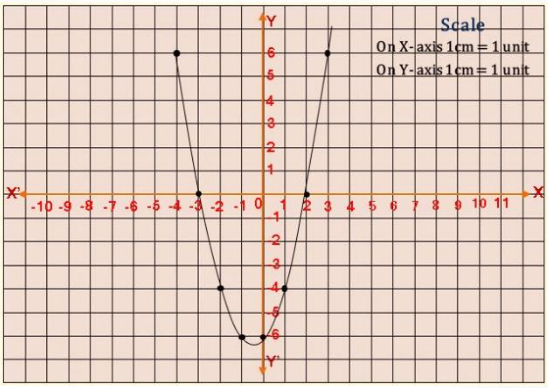
Graphical Representation of Linear polynomial:
$y = x + 2$

x	−3	−2	−1	0	1	2
y = x + 2	−1	0	1	2	3	4
(x, y)	(−3, −1)	(−2, 0)	(−1, 1)	(0, 2)	(1, 3)	(2, 4)



Graphical Representation of Quadratic polynomial:
$y = x^2 + x - 6$

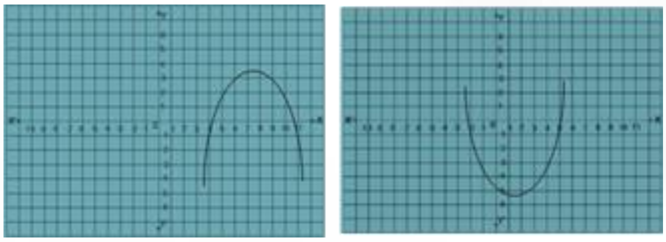
x	−4	−3	−2	−1	0	1	2	3
$y = x^2 + x - 6$	6	0	−4	−6	−6	−4	0	6
(x, y)	(−4, 6)	(−3, 0)	(−2, −4)	(−1, −6)	(0, −6)	(1, −4)	(2, 0)	(3, 6)



- For any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ $(a \neq 0,)$ either opens upwards like ∪ or opens downwards like ∩ This depends on whether $a > 0$ or $a < 0$.
  The shape of these curves is called parabolas

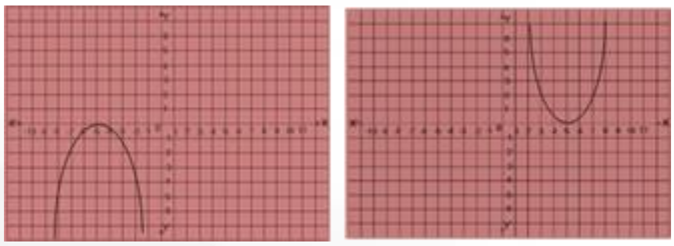
- The shape of the graph of $y = ax^2 + bx + c$, $(a \neq 0)$ the following three cases arise.

Case (i) :



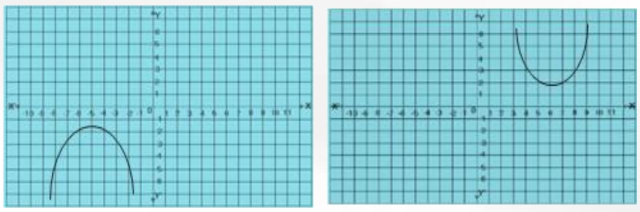
Here, the graph cuts X – axis at two distinct points. In this case, the x-coordinates of those two points are the two zeroes of the quadratic polynomial $ax^2 + bx + c$. The parabola opens either upward or downward.

Case (ii) :



Here, the graph touches X – axis at exactly one point. In this case, the x-coordinate of that point is the only zero for the quadratic polynomial $ax^2 + bx + c$.

Case (iii) :

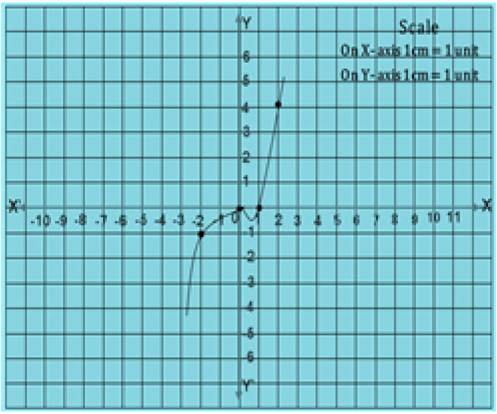


Here, the graph is either completely above the X-axis or completely below the X – axis.
So, it does not cut the X-axis at any point.
The quadratic polynomial $ax^2 + bx + c$ have no zero in this case.

Graphical Representation of Cubic polynomial:

$y = x^3 - x^2$

x	−2	−1	0	1	2
$y = x^3 - x^2$	−12	−2	0	0	4
(x, y)	(−2, −12)	(−1, −2)	(0, 0)	(1, 0)	(2, 4)



- $P(x) = ax + b$ is linear polynomial
  zero of the polynomial is $x = -\frac{b}{a}$
- $P(x) = ax^2 + bx + c\ (a \neq 0)$ is general form of quadratic polynomial.
  Sum of the zeroes $= \alpha + \beta = -\frac{b}{a} = \frac{-(\text{coefficent of } x)}{\text{coefficent of } x^2}$
  Product of the zeroes $= \frac{c}{a} = \frac{\text{constant}}{\text{coefficent of } x^2}$
- $p(x) = ax^3 + bx^2 + cx + d\ (a \neq 0)$ is general form of cubic polynomial.
  $\alpha + \beta + \gamma = -\frac{b}{a} = \frac{-(\text{coefficent of } x^2)}{\text{coefficent of } x^3}$
  $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{\text{coefficent of } x}{\text{coefficent of } x^3}$
  $\alpha\beta\gamma = -\frac{d}{a} = \frac{-(\text{constant})}{\text{coefficent of } x^3}$
- If α, β are the zeroes of quadratic polynomial then its form is $k[x^2 - (\alpha + \beta)x + \alpha\beta]$
- If α, β and γ are the zeroes of the cubic polynomial then its form is
  $k[x^3 - (\alpha + \beta + \gamma)x + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$

Division Algorithm for the polynomials:

If p(x) and g(x) are any two polynomials with $g(x)\ ^1\ 0$, then we can find polynomials

q(x) and r(x) such that $p(x) = g(x) \times q(x) + r(x)$,

Where either $r(x) = 0$ or degree of r(x) < degree of g(x) if $r(x) \neq 0$

We have the following results from the above discussions

(i) If g(x) is a linear polynomial then $r(x) = r$ is a constant.

(ii) If degree of $g(x) = 1$, then degree of $p(x) = 1 +$ degree of q(x).

(iii) If p(x) is divided by $(x - a)$, then the remainder is p(a).

(iv) If $r = 0$, we say q(x) divides p(x) exactly or q(x) is a factor of p(x)

4. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Linear equation in two Variables:

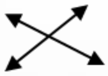
An equation of the form $ax + by + c = o$ where a, b, c are real numbers and $(a^2 + b^2 \neq 0)$ is called linear equation in two variables.

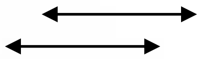
Pair of Linear equation in two Variables:

Two linear equations in two variables of the same type are called a pair of linear equations in two variables.

$a_1x + b_1y + c_1 = 0$ $(a_1^2 + b_1^2 \neq 0)$, $a_2x + b_2 y + c_2 = 0$ $(a_2^2 + b_2^2 \neq 0)$; $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers

When two lines are drawn in the same plane, only one of the following three situations is possible:

i) The two lines may intersect at one point. 

ii) The two lines may not intersect i.e., they are parallel. 

iii) The two lines may be coincident.(actually both are same) 

Let $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ form a pair of linear equations in two variables, then following situations can arise:

Condition	Graphical representation	Algebraic representation	Algebraic interpretation
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solutions	Consistent and Independent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	Consistent and Dependent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	Inconsistent

A pair of linear equations can be solved by three methods

i) Graphical method

ii) Substitution method

iii) Elimination method

iv) Cross multiplication method

5. QUADRATIC EQUATIONS

- General form of quadratic equation is $ax^2 + bx + c = 0$ $(a \neq 0)$.
- If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$

  Sum of the roots $= \alpha + \beta = -\frac{b}{a}$

  Product of the roots $= \frac{c}{a}$

- The quadratic equation formed by the roots α and β is $x^2 - (\alpha + \beta)\,x + \alpha\beta = 0$
- Roots of the quadratic equation $ax^2 + bx + c = 0$ are: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminate:

$b^2 - 4ac$ is called discriminate of quadratic equation is $ax^2 + bx + c = 0$

Nature of the roots:

For $ax^2 + bx + c = 0$

If $b^2 - 4ac > 0$, the roots are real and distinct

If $b^2 - 4ac = 0$, then roots are equal

If $b^2 - 4ac < 0$, then no real roots.

Methods of solving a quadratic equation:

i) By factorization method

ii) By complementing the square method

iii) By quadratic formula

6. PROGRESSIONS

Arithmetic progression (A.P.):

Except first term, all terms are obtained by adding a fixed term to the preceding term. The fixed term is called common difference.

- The terms in A.P are: $a, a + d, a + 2d, \ldots\ldots a + (n - 1)d$
- Common difference $(d) = a_2 - a_1 = a_3 - a_2 = \ldots\ldots = a_n - a_{n-1}$.
- n^{th} term $= a_n = a + (n - 1)\,d$.
- Sum of the n terms $= S_n = \frac{n}{2}[2a + (n - 1)d]$

  $= \frac{n}{2}[a + l]$ where a is first term, l is last term.

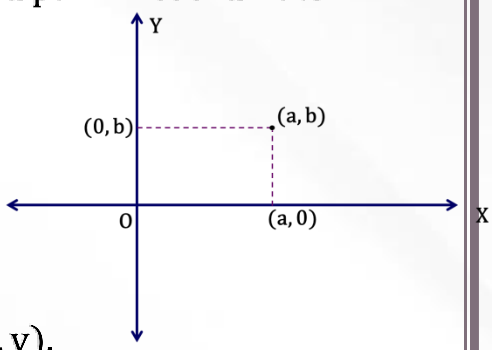
Geometric progression (G.P.):

Except first term, all terms are obtained by multiplying a fixed term to the preceding term. The fixed term is called common ratio.

- The terms in G.P are: a, ar, ar^2, ... ar^{n-1}
- Common ratio (r) = $\frac{a_2}{a_1} = \frac{a_3}{a_2} = ... = \frac{a_n}{a_{n-1}}$. n^{th} term= $a_n = a.r^{n-1}$

7. COORDINATE GEOMETRY

- For any two real numbers 'a' and 'b', (a, b) is called ordered pair. In coordinate geometry, it is called as coordinate point.
  In a point (a, b)
  a is called as 'abscissa' or 'x – coordinate'
  b is called as 'ordinate' or 'y – coordinate'



- If (x , y) = (a, b) , then x = a and y = b
- Point of intersection of axes(x and y) is origin i.e., (0, 0)
- Any point on x – axis (x, 0) and any point on y – axis is (0, y).

Distance between Two Points:

Distance between two points on X - axis and on a line parallel to X - axis:

- Distance between any two points $A(x_1, 0)$ and $B(x_2, 0)$ on x – axis is $|x_2 - x_1|$
- Distance between any two points $A(x_1, 0)$ and $B(x_2, 0)$ on a line parallel to x – axis is $|x_2 - x_1|$

Distance between two points on X- axis and on a line parallel to X axis:

- Distance be1tween any two points $A(0, y_1)$ and $B(0, y_2)$ on y – axis is $|y_2 - y_1|$
- Distance be1tween any two points $A(0, y_1)$ and $B(0, y_2)$ on a line parallel to y – axis is $|y_2 - y_1|$

Distance between two points anywhere on the coordinate plane:

- Distance between any two points $A(x_1, y_1)$ and B (x_2, y_2) is
  $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Distance between origin[(0, 0)] and a point P(x, y) is OP= $\sqrt{x^2 + y^2}$

Section formula:

If a point P (x, y) divides the line segment joining the points A(x_1, y_1) and B (x_2, y_2) internally in the ratio $m_1 : m_2$, then P (x, y) = $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

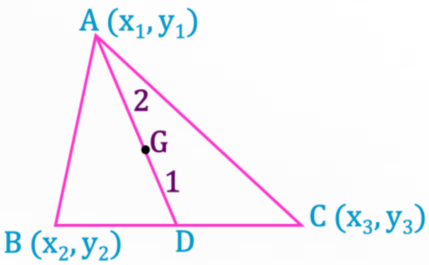
- If a point P (x, y) divides the line segment joining the points A(x_1, y_1) and B (x_2, y_2) internally in the ratio k : 1, then P (x, y) = $\left(\frac{kx_2 + x_1}{k + 1}, \frac{ky_2 + y_1}{k + 1}\right)$

Midpoint of line segment:

The midpoint of a line segment joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Centroid of the triangle:

The point of concurrence of medians of a triangle is called centrefold of the triangle. It is denoted by G. G divides median in the ratio 2 : 1.



If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of a triangle ABC, then its centroid is

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

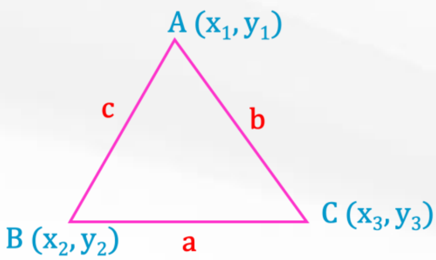
Points of trisection:

The points P, Q on a line segment AB are said to be the points of trisection, if P and Q divides AB into three equal parts i.e., AP = PQ = QB and they divides $\overline{AB}$ in the ratio 1 : 2 and 2 : 1 respectively.

Area of the triangle:

If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of a triangle ABC, then its area is

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$



Heron's formula :

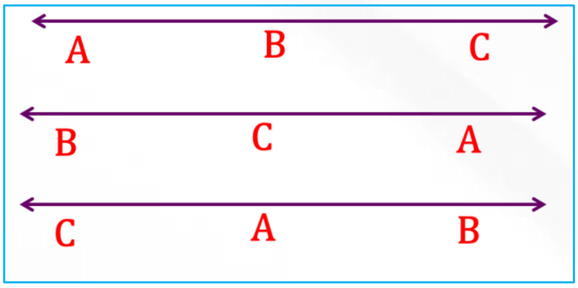
- Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of a ΔABC, then AB = c, BC = a and AC = b

  Area of Δ ABC = $\sqrt{s(s - a)(s - b)(s - c)}$ where $s = \frac{a+b+c}{2}$

Collinear points:

The points which are lie on the same line are called collinear points.

A, B and C said to be collinear if AB + BC = AC or BC + CA = AB or AB + AC = BC



- Area of $\Delta ABC = 0 \Rightarrow \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$, then the points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear points
- If slope of AB = slope BC = slope of AC, then A, B and C are collinear.

Slope of a line:

A line makes an angle θ with the positive direction of x – axis, and then tan θ is called the slope of the line. It is denoted by m.

- The slope of the line joining the points $A(x_1, y_1), B(x_2, y_2)$ is $m = \tan\theta = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$.

8. SIMILAR TRIANGLES

Similar figures:

The geometrical figures, which have same shape but not in size, are called similar figures.

- All regular polygons of the same no. of sides are similar.
- Two congruent figures are similar but two similar figures need not be congruent.

Similar triangles:

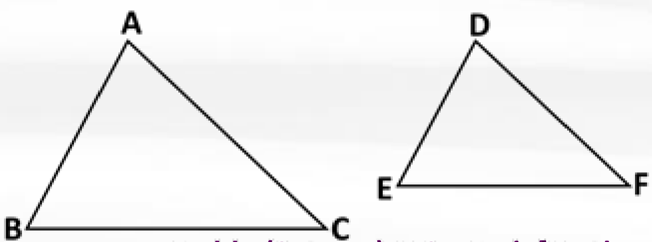
Two triangles are said to be similar If :

(i) their corresponding angles are equal and

(ii) their corresponding sides are in proportional (in same ratio).

- If $\Delta ABC \sim \Delta DEF$, then (i) $\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$

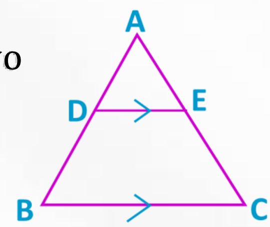
$$\text{(ii)} \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = k$$



- Here k is scale factor,

  (i) If $k > 1$, then we get enlarged figures
  (ii) If $k = 1$, then we get congruent figures
  (iii) If $k < 1$, then we get reduced figures.

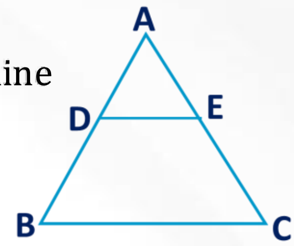
Basic proportionality theorem (Tale's theorem):



If a line drawn parallel to one side of a triangle intersecting the other two sides at distinct points, then the other two sides are divided in the same ratio.

In ΔABC, if $DE \parallel BC$ then $\frac{AD}{DB} = \frac{AE}{EC}$

Converse of Basic proportionality theorem:

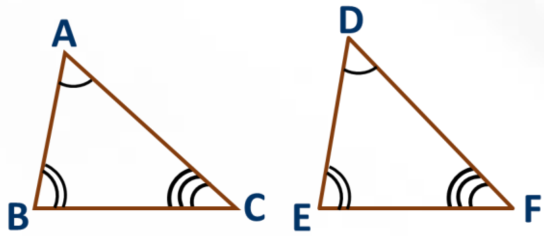


If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

In ΔABC $\frac{AD}{DB} = \frac{AE}{EC}$, then $DE \parallel BC$

Criterion of similar triangles:

A.A.A Similarity:

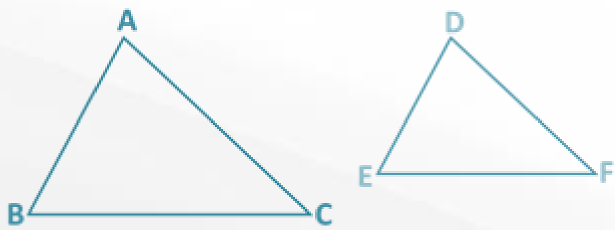


In two triangles, if corresponding angles are equal, then their corresponding sides are in proportion and hence the triangles are similar.

- In ΔABC and ΔDEF

  If $\angle A = \angle D; \angle B = \angle E ; \angle C = \angle F$, then $\Delta ABC \sim \Delta DEF$

S.S.S Similarity:

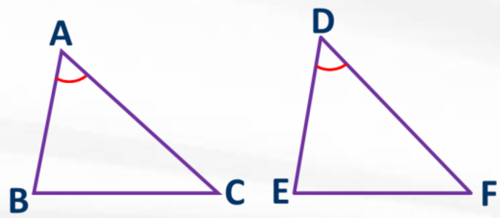


In two triangles, if corresponding sides are proportional, then their corresponding angles are equal and hence the triangles are similar

- In ΔABC and ΔDEF

  If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ then $\Delta ABC \sim \Delta DEF$

S.A.S Similarity:



If one angle of a triangle is equal to one angle of the other triangle and the including sides of these angles are proportional, then the two triangles are similar.

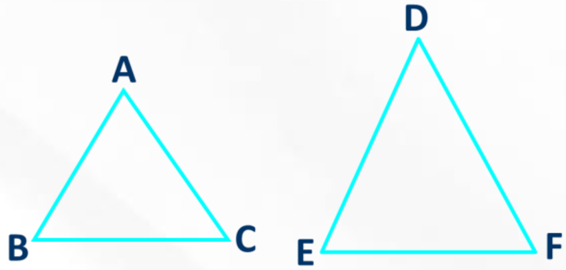
- In ΔABC and ΔDEF

  If $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$ then $\Delta ABC \sim \Delta DEF$

- The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

  $\Delta ABC \sim \Delta DE$

  $$\frac{ar\,(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$



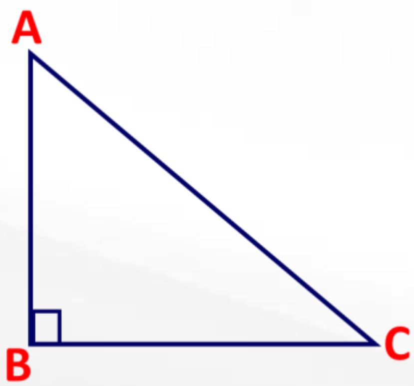
- The ratio of areas of two similar triangles is equal to the ratio of the square of their corresponding sides.
- The ratio of perimeters of two similar triangles is equal to the ratio of their corresponding sides.

Pythagoras theorem:

In a right-angled triangle square of hypotenuse is equal to sum of the squares of two sides.

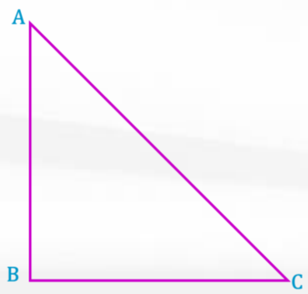
$(\text{Hypotenuse})^2 = (\text{side})^2 + (\text{side})^2$

In ΔABC, $\angle B = 90^0$, then $AC^2 = AB^2 + BC^2$



Converse of Pythagoras theorem:

In a triangle square of one side is equal to sum of the squares of other two sides, then it is a right angled triangle

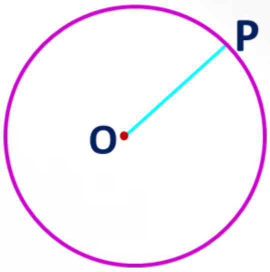


In ΔABC, $AC^2 = AB^2 + BC^2$, then $\angle B = 90^0$,

9. TANGENTS & SECANTS TO A CIRCLE

Circle:

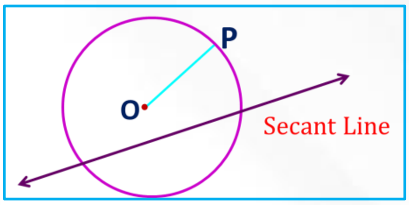
Set points which are moving at a constant distance from a fixed point is known as a circle.



Constant distance is called radius of the circle and fixed point is centre of the circle.

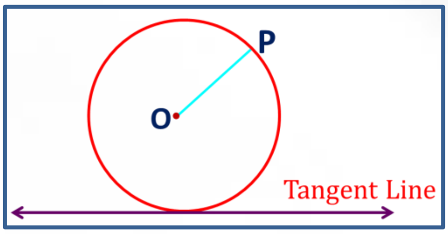
Secant line:

A line which is passing through any two points on the circle is called secant line.

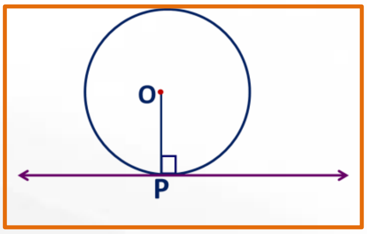


Tangent line:

A line which touches at only one point on the circle is called tangent and that point is called point of contact.



- Angle between radius and tangent at point of contact is 90^0.



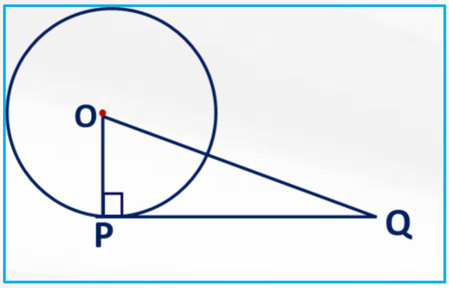
- We can draw infinitely many tangents to a circle.

Finding the length of the tangent to a Circle:

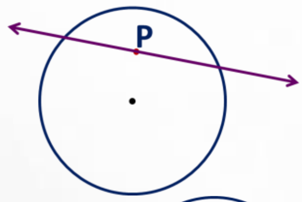
In ΔOPQ, $\angle P = 90^0$, then $OQ^2 = OP^2 + PQ^2$

$$PQ^2 = OQ^2 - OP^2$$

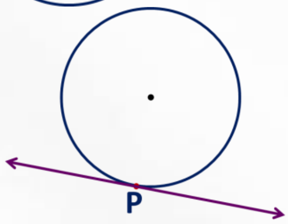
$$PQ = \sqrt{OQ^2 - OP^2}$$



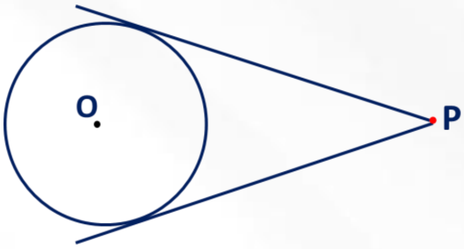
Number of tangents to a Circle from any Point:



Case 1: There is no tangent to a circle passing through a point inside the circle



Case 2: There is one and only one tangent to a circle at a point on the circle



Case 3: There are exactly two tangents to a circle through a point outside the circle.

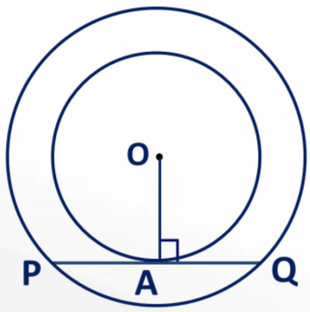
- Two tangents drawn from external point to a circle are equal in length.

Note: **(i)** The word 'tangent' comes from the Latin word 'tangere', which means to touch and was introduced by Danish mathematician Thomas Fineke in 1583.

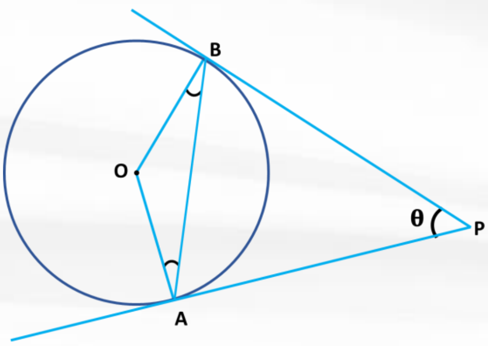
(ii) The line containing the radius through the point of contact is also called the 'normal to the circle at the point'.

- In two concentric circles, the chord of the bigger circle that touches the smaller circle is bisected at the point of contact with the smaller circle

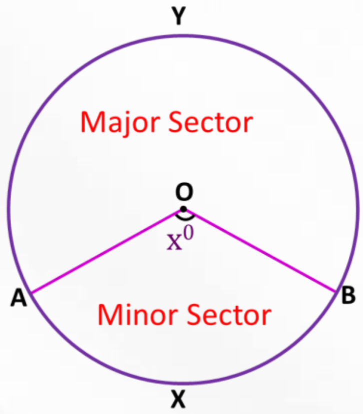
  PA = AQ



- If two tangents AP and AQ are drawn to a circle with centre O from an external point A, then $\angle APB = 2\angle OAB = 2\angle OBA$



Sector of a Circle:

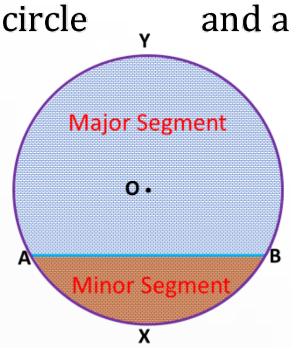


Part of a circle made of the arc of the circle along with its two radii

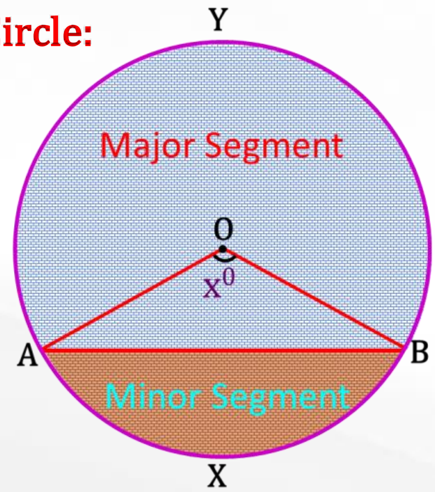
- Area of sector OAXB $= \frac{x^{\circ}}{360^{\circ}} \pi r^2$.
- Area of sector OAYB $= \pi r^2 - \frac{x^{\circ}}{360^{\circ}} \pi r^2$.

Segment of a Circle:

The area enclosed by an arc of a circle and a chord is called segment of the circle

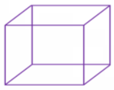
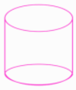


Finding area of Segment of a Circle:



- Area of Minor segment OAXB = Area of sector OAXB – Area of ΔOAB
- Area of Major Segment OAYB = Area of Circle – Area of sector OAXB
- A line segment joining any two points on circle is called chord of the circle.
- The longest chord, which is passing through the centre of the circle, is called diameter.

10. MENSURATION

NAME	SHAPE	L.S.A/ C.S.A(sq.units)	T.S.A (sq. units)	VOLUME (cubic units)
Cube		$4\ a^2$	$6\ a^2$	a^3
Cuboid		$2h\ (a + b)$	$2(lb + bh + hl)$	lbh
Cylinder		$2\pi\ r\ h$	$2\pi r(r + h)$	$\pi\ r^2\ h$
Cone		$\pi\ r\ l$	$\pi r(l + r)$	$\frac{1}{3}\pi\ r^2 h$
sphere		$4\pi\ r^2$	$4\pi r^2$	$\frac{4}{3}\pi\ r^3$
Hemi- sphere		$2\pi\ r^2$	$3\pi r^2$	$\frac{2}{3}\pi\ r^3$

Surface area of the Combination of the Solids:

TSA of new solid = CSA of one hemisphere + CSA of cylinder + CSA of other hemisphere

Volume of the Combination of the Solids:

The volume of the solid formed by joining two or more basic solids is the sum of the volumes of the constituents.

- When a solid converts from one shape to another shape, there is no change in volume.

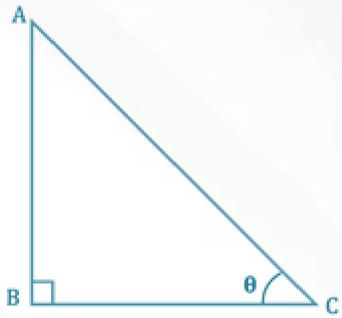
11. TRIGONOMETRY

The word Trigonometry derived from Greek word,

tri → three, gonia → angle and metron → to measure

Angle: The figure formed by two rays meeting at a common end point is an angle.

Naming the sides in a right-angled triangle:



AB is opposite side of θ (opp)
BC is adjacent side of θ(adj)
AC is hypotenuse (hyp)

$$(\text{hyp})^2 = (\text{opp})^2 + (\text{adj})^2$$

Trigonometric ratios:

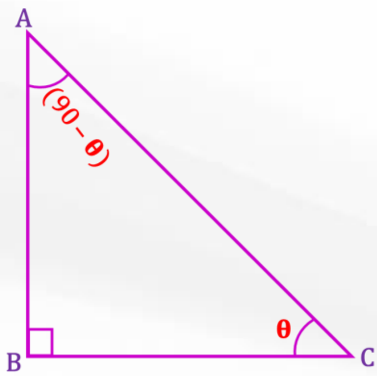
$$\sin \boldsymbol{\theta} = \frac{\text{Opposite side of } \theta}{\text{Hypotenus}} \qquad \text{cosec}\, \boldsymbol{\theta} = \frac{\text{Hypotenus}}{\text{Opposite side of } \theta}$$

$$\cos \boldsymbol{\theta} = \frac{\text{Adjacent side of } \theta}{\text{Hypotenus}} \qquad \sec \boldsymbol{\theta} = \frac{\text{Hypotenus}}{\text{Adjacent side of } \theta}$$

$$\tan \boldsymbol{\theta} = \frac{\text{Opposite side of } \theta}{\text{Adjacent side of } \theta} \qquad \cot \boldsymbol{\theta} = \frac{\text{Adjacent side of } \theta}{\text{Opposite side of } \theta},$$

Complimentary Angles:

Two angles are said to be complimentary, if their sum is 90^0.



$$\sin \theta = \frac{AB}{AC} \qquad \sin (90 - \theta) = \frac{BC}{AC}$$

$$\cos \theta = \frac{BC}{AC} \qquad \cos (90 - \theta) = \frac{AB}{AC}$$

$$\tan \theta = \frac{AB}{BC} \qquad \tan (90 - \theta) = \frac{BC}{AB}$$

$$\cot \theta = \frac{BC}{AB} \qquad \cot (90 - \theta) = \frac{AB}{BC}$$

$$\text{cosec}\, \theta = \frac{AC}{AB} \qquad \text{cosec}\, (90 - \theta) = \frac{AC}{BC}$$

$$\sec \theta = \frac{AC}{BC} \qquad \sec (90 - \theta) = \frac{AC}{AB}$$

$\sin (90 - \boldsymbol{\theta}) = \cos \boldsymbol{\theta}$, $\text{cosec}\, (90 - \boldsymbol{\theta}) = \sec \boldsymbol{\theta}$, $\tan (90 - \boldsymbol{\theta}) = \cot \boldsymbol{\theta}$,

$\cos (90 - \boldsymbol{\theta}) = \sin \boldsymbol{\theta}$, $\sec (90 - \boldsymbol{\theta}) = \text{cosec}\, \boldsymbol{\theta}$, $\cot (90 - \boldsymbol{\theta}) = \tan \boldsymbol{\theta}$,

Trigonometric Identities:

- $\sin^2\theta + \cos^2\theta = 1$
  $\sin^2\theta = 1 - \cos^2\theta$
  $\cos^2\theta = 1 - \sin^2\theta$
- $\sec^2\theta - \tan^2\theta = 1$
  $\sec^2\theta = \tan^2\theta + 1$
- $\text{cosec}^2\theta - \cot^2\theta = 1$
  $\sec^2\theta - 1 = \tan^2\theta.$

$\sec^2\theta - \tan^2\theta = 1$
$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$
$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$

$\text{cosec}^2\theta - \cot^2\theta = 1$
$(\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta) = 1$
$\text{cosec}\,\theta + \cot\theta = \frac{1}{\text{cosec}\,\theta - \cot\theta}$
$\text{cosec}\,\theta - \cot\theta = \frac{1}{\text{cosec}\,\theta + \cot\theta}$

- $\sin\theta \times \text{cosec}\,\theta = 1$
  $\cos\theta \times \sec\theta = 1$
  $\tan\theta \times \cot\theta = 1$

Trigonometric Ratios of some Specific Angles:

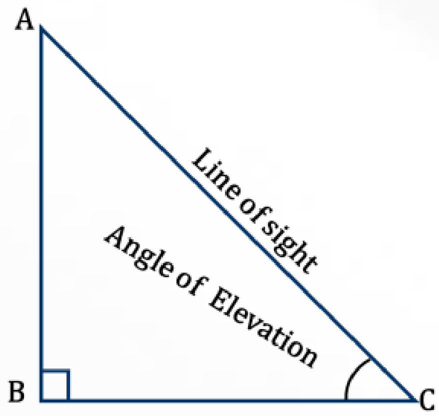
	0^0	30^0	45^0	60^0	90^0
sin **θ**	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos **θ**	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan **θ**	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cot **θ**	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec **θ**	Not defined	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	0
cosec **θ**	0	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	Not defined

12. APPLICATION OF TRIGONOMETRY

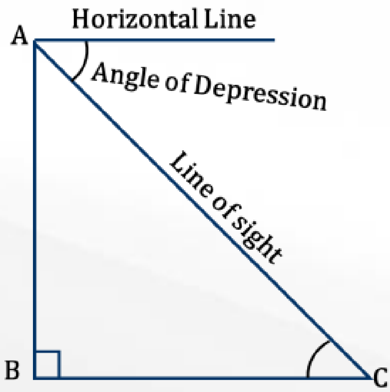
Horizontal line: A line which is parallel to earth from observation point to object is horizontal line

Line of sight: The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

Angle of elevation: the line of sight is above the horizontal line and angle between the line of sight and horizontal line is called angle of elevation.



Angle of depression: the line of sight is below the horizontal line and angle between the line of sight and horizontal line is called angle of depression.



Solving procedure:

- All the objects such as tower, trees, buildings, ships, mountains etc. shall be consider as linear for mathematical convenience.
- The angle of elevation or angle of depression is considered with reference to the horizontal line.
- The height of observer neglected, if it is not given in the problem.
- To find heights and distances we need to draw figures and with the help of these figures we can solve the problems.

13. PROBABILITY

- J Cordon Italian mathematician wrote the first book on probability named "the book of games and chance".
- Experimental or empirical probability = $\frac{\text{no.of trials in which event happened}}{\text{total no.of trials}}$
- Classical or theoretical probability = $\frac{\text{no.of trials in which event happened}}{\text{total no.of trials}}$

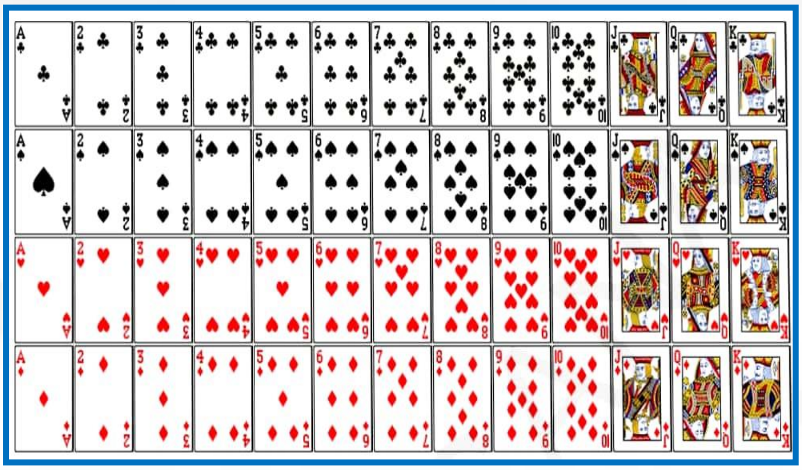
Some words in probability:

Experiment: A repeatable procedure with a set of possible results.
Outcome: a possible result of an experiment.
Sample space : All the possible outcomes of an experiment.
Sample point : Just one of the possible outcome.
Event : One or more outcomes of an experiment.
Equally likely events : Each event of an experiment occurs with equal probability.
Impossible event : If there is no probability of an event to occur then it is impossible event. Its probability is zero.
Sure or certain event : If the probability of an event is 1 then it is sure or certain event.
Complimentary event : Let E denote the event, ' not E' is called complimentary event of E. It is denoted by $\overline{E}$.

- $P(\overline{E}) = 1 - P(E) \Rightarrow P(\overline{E}) + P(E) = 1$
- $0 \leq P(E) \leq 1$

Deck of cards: A deck of playing cards consists of 52 cards which are divided into 4 suites of 13 cards each. They are black spade ♠, black clubs ♣, red heart ♥ and red diamond ♦ .

- The cards in each suit are: 2, 3, 4, 5, 6, 7, 8, 9, 10, Ace, Jack, Queen and King.
- Jack, Queen and King are called face (picture) cards.



	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

14. STATISTICS

The three measures of central tendency are: (i) Mean (ii) median (iii) mode

Mean

If $x_1, x_2, \ldots, x_n$ are n observations then mean is $\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\sum x_i}{n}$

If $x_1, x_2, \ldots, x_n$ are of n observations occurs $f_1, f_2, \ldots, f_n$ times respectively then mean is

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + f_3 x_3 + \cdots + f_n x_n}{f_1 + f_2 + f_3 + \ldots + f_n} = \frac{\sum f_i x_i}{\sum f_i}$$

Class mark (mid value): $x = \frac{\text{lowr limit} + \text{upper limit}}{2}$

Methods of finding mean:

Direct method: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$; x_i is class mark of i^{th} class, f_i is frequency of class.

Assumed mean method: $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$; $d_i = x_i - a$ and a is assumed mean.

Step – deviation method: $\bar{x} = a + \left(\frac{\sum f_i \mu_i}{\sum f_i}\right) h$; $\mu_I = \frac{xi - a}{h}$, h is class size.

Median

Median is the middle most observation of given data.

For un grouped data:

First, we arrange given 'n' observations into ascending or descending order.

If n is odd median $= \left(\frac{n+1}{2}\right)^{th}$ observation.

If n is even median $= \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2} + 1\right)^{th} \text{observation}\right]$

For grouped data:

Median $= l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times h$, where

l = lower boundary of the median class

c.f = cumulative frequency of the class preceding the median

n = number of observations

f = frequency of the median class

h = width of the class or Class size

Mode

A mode is that value among the observations which occurs most frequently.

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$, where

l = lower boundary of the modal class

F_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

h = width of the class or Class size

Empirical relation: mode = 3(median) – 2(mean)